Effective Questioning and Responding in the Mathematics Classroom[1]

John Mason
Open University & University of Oxford
2010

0. Introduction

Asking learners (especially children) questions is so strongly embedded in our culture that most adults do it when in the company of children, and most children do it when playing 'school'. Furthermore, in these types of interactions, the questioner usually knows the answer, and most children quickly work out that this is the case. Questions in school are seen as some sort of testing process, through which learners supposedly learn, and this carries over into adult-child interactions. An extreme form is the *cloze* technique of pausing and expecting students to fill in the missing word. Many classroom interactions are some variant on "guess what is in my mind". By contrast, adults are more likely to ask each other genuine information-seeking, genuinely enquiring. How do questions arise in the classroom? How can we use them effectively? How can we stimulate learners to ask their own questions? These issues are addressed through a number of conjectures which cannot be proved as universal, but which can be tested in your own experience.

1. How Do Questions Arise?

Not all utterances with a question mark are questions, and some statements are intended to produce a response. For example, "We don't do that in here, do we?" is an assertion not a question, and "Tell me what you are thinking", or "Tell me what you have been doing" require or expect a response. For ease of reference 'question' will be taken to include any utterance (or gesture or posture) which expects a response. So how do questions arise, particularly in an educational context?

> *Conjecture*: an adult asks a learner a question when the adult, while in the presence of the learner, experiences a shift in the focus of their own attention. The question is intended to reproduce that shift of focus in the learner.
>
> In particular, enquiry–questions are asked when people become aware that they are uncertain, confused, stuck, struck by something they cannot account for, or when they realise that some expectation is being contradicted.

This conjecture has to be tested in your own experience, through trying to catch yourself suddenly asking children or other learners some question. Then ask yourself, where did that question come from? What was the impulse to ask? For example, here are some situations I have caught myself in:

> I am with a child, I notice something, and I experience a desire that the child see it also; I find myself asking a pointed or focusing question;
>
> A technical term comes to mind that the learner is supposed to know so I ask what the term means;
>
> I become aware of a logical consequence of something I was thinking about so I ask a pointed question of the form "which means that …" or something similar;

[1] This is a revised version of a chapter that first appeared in Mason (2002). I am very grateful to Hilary Povey for suggesting that I rework these ideas.

I become aware that some situation is a particular case of a more general phenomenon, theme, technique, etc., so I ask a pointed question, varying from "does that always happen/work?" to "so what happens in general";

These experiences suggest that questions often arise when I experience a contrast, or a change in my focus, and without even being aware of it, I use the format of a question to try to direct learners' attention. Furthermore, and this is the important part, I often find that it is only when I hear the learner's response that I am aware that it conflicts with what I was expecting. The response shifts my attention from dwelling in my own focus of attention, to recognising that I had something in my head, some expectation. That makes me aware that I have asked a question with a specific answer in mind. Before that, I am immersed in the flow of my own attention.

Questions such as "How did we do this last week?", "What is this diagram saying?", "Why did you …", and so on, all try to focus attention on something that I believe is being overlooked or needs stressing. Even when I am using an enquiry-question because there is something I don't know, the question itself arises because of a sense of gap or uncertainty in my own mind. The question format comes naturally, if tentatively to focus others' attention on my problem. My enculturation into social practices means that I prefer an indirect question to a direct admission that I don't know something.

Thus it seems that questions have the effect of focusing or directing other peoples' attention. They arise from the flow of attention of the asker, and they are likely to be a disturbance to the flow of other people's attention. Unfortunately, that disturbance may not always be welcome.

1.1 Controlling Questions

Because it is an accepted cultural norm that questions are supposed to be answered, questioning is one way in which people exert social control, one way in which they assert authority or power (Ainley 1987, Love & Mason 1992). For example "What do we do when we come into the classroom …?", "We don't do that, now do we?", "Where do we put the equals sign?". This applies especially in a class, where by picking on certain individuals to respond, and by stopping one line of discussion through introducing a new one, the teacher retains control. A natural and frequent occurrence however, is that not-answering is used by learners as a form of reaction or revolt, even an attempt to grab back some power and influence. This is most likely when learners feel buffeted by questions, or when they detect that questions are being used for control purposes.

Similarly, we retain control over learner attention by asking focusing questions such as "What is in front of the x?", "What is next to the three?", "What do we do with the variable?", "What does the diagram tell us?", and more generally, "What did we/you do last time?", "Can you give me an example?", "Have you seen something like this before?", "What does it say in the question?", "What do you know and what do you want?". Sometimes the question usefully redirects attention and the learner is able to take back the initiative, but in many cases, if the learner knew the answer to the question, they would probably not be stuck so the question would not have to be asked in the first place! Yet somehow we naturally ask the question.

1.2 Cloze Technique

Pausing in a flow of statements and expecting students to fill in the missing word is a common format for testing-questions in classrooms. For example, "This shape is called a ____.", and "The next thing we do is to carry down the ___." Note that the missing word is usually at the end of a sentence. The idea is that students are having their attention directed to the key detail. The production by them of the appropriate label is supposed to reinforce memory so that they will know what to do next time. It is presumed that learners are rehearsing patterns of inner speech which will help them carry out the technique.

However, there is effective use and ineffective use of this technique, and the two are rather hard to tell apart. If you listen to a lesson in which there is a lot of this going on, you will soon see that children can chorus out an expected word without knowing anything of what is going on because the reasoning is being done by the teacher, and the missing word becomes clear even if you do not know what is going on. Far from rehearsing the useful inner incantations of a technique, students are only called upon to fill in a technical term. On the other hand, carrying out a technique or method can be supported through an inner commentary (not necessarily either voiced or even sub-vocal), but if learners are unaware of this possibility, they may be trying to memorise actions without using mental imagery or inner monologue to support them! When (not if!) you catch yourself pausing and expecting children to complete your statements, make sure that you are getting them to fill in the reasoning, rehearsing the commentary as a whole, not simply parroting technical terms.

1.3 Genuine-enquiry

Not all questions exert control explicitly. For example it is possible to enquire genuinely about what someone is thinking: "How did you get that?", "Why did you add these two numbers?", "Can you tell me how to do this type of question in the future?". Of course the respondent may interpret the question as an indication that there is something wrong and that the questioner knows this and even knows what it should be. The fact that it is being asked by a teacher is likely to lead the learner into believing that the teacher knows the answer and expects the learner to know it too, and-or that what the learner has been doing is not correct or not appropriate. Thus the fact of a question being asked is likely to generate a defensive stance. Voice tones together with posture and gesture can be critical for indicating genuineness. A slight change of inflection, a suitable pause and facial movement can make all the difference. It takes time to build up trust and to establish a suitable mathematical environment, what later is referred to as a 'conjecturing atmosphere'.

1.4 Meta-questions

Meta-questions are questions about the activity which draw learner attention out of the particularities of the current task with a view to making them aware of a process. For example: "What would you have to do next time to answer a similar question?", "What led you to choose this approach?", "What question am I going to ask you?" are typical meta-questions.

This last question is typical of a range of increasingly indirect prompts used to encourage learners to internalise questions which they could usefully ask themselves. When a particular type of question is proving fruitful such as "Can you give me an example", or "What do you Know in this problem, and what do you Want to find?", the teacher can explicitly refer to the use of these questions, perhaps by asking themselves out loud and replying in front of the learners while working on a problem, then using them with learners.

If learners come to rely on the teacher to ask the same question every time, then learners are being trained in dependency, not educated. After a period of time it is important to become less and less direct, and more and more indirect so that learners begin to internalise the question. The aim is that they take the initiative to ask themselves. To do this they need to withdraw from immediate activity and reflect on it 'as if from another dimension' (geometrically, a reflection can only be manifested if there is a move into a higher dimension). Eventually you can ask questions like "What question do you think I am going to ask you?". Of course the first time you ask this they will probably not know what you are asking, but you can tell them, then use the same prompt again later.

Teachers have been known to put up a poster with a few pertinent questions listed. But eventually the poster must come down (or be replaced with a fresh one). If the poster remains up all term or all year, then learners are likely to become dependent on it. If the teacher has to keep asking the same questions, the learners are not being educated. By obscuring the poster after a while and referring to it indirectly, then later removing it, and by using more and more indirect prompts (such as meta-questions), learners can be induced to incorporate those questions into their way of thinking. That frees you to make use of a further collection of additional questions.

The process of moving from *directed questions*, through increasingly *indirect prompts* towards *spontaneous use* by learners is also known as scaffolding (the direct questions) and fading (increasingly indirect prompts). The term *scaffolding* was introduced by Wood, Bruner & Ross (1976) and used by Bruner (1986) to bring ideas of the Russian psychologist Lev Vygotsky to the West. The effectiveness of scaffolding lies not in the actual scaffolding but in the fading, the increasingly indirect prompts so that learners internalise the support (Brown, Collins & Duguid 1989).

1.5 Open and Closed Questions

There is a penchant for classifying questions as being open or closed, or more specifically, open-ended or open-fronted, and closed-ended or closed-fronted. For example

> "What is a triangle with three equal sides called?" and "1/3 – 1/4 = ?" are clearly closed at both ends, because what has to be done is specified, and there is a single correct answer;
>
> "Explore the relationship between a polygon being equi-angular and equi-lateral" and "What fractions can be the difference of two unit fractions?" are open-fronted, because the learners have to decide what they are actually going to work on, and perhaps open-ended because there is no specific well known answer to be found;
>
> "For which polygons does equilateral imply equiangular and vice versa?" and "In how many ways can a given unit fraction be the difference of two unit fractions?" are fairly open-fronted because the learners have to decide what polygons or fractions to work on, but are closed-ended because there are definite and known answers.
>
> "What can you tell me about this shape?" or "What do you notice?" is open-ended but closed-fronted because the shape or object is specified but the features the learner chooses to stress and express are not, though it is also likely to be received as "Guess what is in my mind".

However questions are just words with a question mark: the notion of openness and closed-ness is more to do with how the question is interpreted than with the question itself. Thus "What is a triangle with three equal sides called?" could be taken as a stimulus to explore the use of the term *equilateral* for other polygons, while "For which polygons does equilateral imply equiangular and vice versa?" could be taken as an instruction to locate and prove a theorem concerning triangles. Thus qualities of openness and closed-ness are in the eye of the beholder. Deliberately placing a particular value on openness and closed-ness, whether at the front or the end, over simply creates an obstacle to exploiting the strengths of each, and appreciating the possible varieties.

2. Using Questions Effectively

Questioning is effective if it contributes to focusing learner attention appropriately. For example, questioning (in its broadest sense) can focus attention on some mathematical possibilities, whether through shifting attention onto a particular detail, relationship or property. It can also draw the learner out of immersion in activity so that attention is directed to the kinds of prompts and questions the teacher is using, so that learners become aware that they could be using those prompts for themselves in the future.

2.1 Interrogating your own experience

The first thing necessary is to try to catch yourself using questions, to try to check any conjectures out for yourself. If you find some agreement with them, if it helps make sense of your past experience, then you may want to work at changing the way you use questions. The rest of this section makes suggestions to this end. If you do not agree with the conjectures, then the rest of this section may provide further food for thought and experimentation. The specific questions are a matter of personal taste and current concerns; the general thrust and ways of working in which they are embedded are what matter.

2.2 Reducing the use of questions for controlling

The conjectures put forward imply that all questions asked by a teacher are controlling to some extent, and certainly intended to disturb the learner's flow (or stuck) thoughts. But it is possible to reduce the use of questions for social control and for exerting authority, so as to allow questions to be used for teaching mathematics.

To locate a question asked for the purposes of social control, ask yourself how you would feel if the learners asked the same question of you! What sorts of questions from a learner would be acceptable and what kinds would be seen as impertinent? The impertinent ones are probably the ones used for controlling and norming. The use of 'we' is also characteristic, and can be used to catch yourself asking this form of question: when you find yourself using 'we', stop and ask yourself who the 'we' is. Notice also that when a teacher reports that in a lesson 'we discussed …' there is no evidence to distinguish between a norming and controlling sequence of questions and a genuine discussion or enquiry. As a form of interaction, controlling and norming questions are perfectly natural, common and necessary, but they may get in the way of developing a conjecturing, enquiring atmosphere in the classroom. There are other equally effective ways of socialising and controlling learners, such as by making a direct instruction or statement.

Where maintaining the power structure is necessary, try using assertions rather than questions. Learners quickly recognise that questions are being used for control purposes, and it merely muddies the water for creating a questioning, conjecturing atmosphere in the classroom which supports rather than obstructs mathematical thinking.

2.3 Funneling

Asking a learner a question is one thing, but what happens if they do not respond? Perhaps the question is too difficult? Perhaps a more pointed, more focused, more precise question will make it clear? So begins a process of funnelling (Bauersfeld 1995, Wood 1998), of playing the game "Guess what is in my mind". The teacher keeps asking one or more learners more and more precise and detailed questions in an attempt to find something that they can answer. John Holt (1964) gave a paradigmatic example of funnelling:

> I remember the day not long ago when Ruth opened my eyes. We had been doing math, and I was pleased with myself because, instead of telling her answers and showing her how to do problems, I was "making her think" by asking her questions. It was slow work. Question after question met only silence. She said nothing, did nothing, just sat and looked at me through those glasses, and waited. Each time, I had to think of a question easier and more pointed than the last, until I finally found one so easy that she would feel safe in answering it. So we inched our way along until suddenly, looking at her as I waited for an answer to a question, I saw with a start that she was not at all puzzled by what I had asked her. In fact she was not even thinking about it. She was coolly appraising me, weighing my patience, waiting for the next, sure-to-be-easier question. I thought "I've been had!" The girl had learned how to make me do the work for her, just as she had learned to make all her previous teachers do the same thing. If I

> wouldn't tell her the answers, very well, she would just let me question her right up to them. p24-25

How can you break out of a funnelling sequence? As soon as you become aware that you are playing some form of "Guess what's in my mind" you have the option of admitting to yourself, or even to them, that you do indeed have something in mind. You can go to one extreme, perhaps, and play a quick game of hangman as you indicate the length of the word you are looking for; at the other extreme you can simply tell them the answer, and then perhaps genuinely enquire why they had not thought of that themselves, and how they might learn to think of it in a similar situation in the future.

2.4 Creating a Conjecturing Atmosphere

A mathematical, or conjecturing atmosphere, is one in which whatever is said is said tentatively as a conjecture in the hope of getting feedback and suggestions for modification. Those who are confident they 'know the answer' tend to keep quiet, or perhaps ask pointed questions in order to support and assist others, while those who are uncertain take every opportunity to say what they can say, and then get help in extending or completing it. Struggle is valued, even praised. No one says "no that's wrong", they say "I invite you to modify your conjecture".

How might a conjecturing atmosphere be developed? The first essential feature is to adopt a conjecturing stance yourself, treating everything said by you or by others as a conjecture which may require modification, not as an assertion that has to be right or wrong. Secondly, take opportunities to praise learners for changing their mind, for modifying what they previously said or did. Thirdly, take opportunities to praise learners for making a conjecture (without implying judgement about the quality or aptness of the conjecture). This enables you to attend to the process and ethos of the topic development rather than to the correctness or otherwise of what is said. Try to put the onus on learners to test out what others conjecture. Thirdly, especially at the beginning, label conjectures as such (someone asserts something forcefully, and you say "conjecture"; someone says "no" or "that's not right" to someone else, and you invite them to change what they say to "I disagree with your conjecture" or "I invite you to modify your conjecture").

Earlier I suggested that questions cause a disturbance to learners, and that not all disturbances may be welcome. In a conjecturing atmosphere learners are confident to contribute because they know they are learning. In an atmosphere of questions which constantly test whether learners know facts, learners may display signs of anxiety (Anderson & Boylan 2000): if questions are perceived as too simple, there may be anxiety about not getting them correct; if questions of different difficulty are offered to learners in accordance with what the teacher thinks they can do, there may be embarrassment; and a learner who answers a hard question may be disliked by peers for whom it is too difficult. Moreover, teacher assessment of what a learner is capable of is one of the principle obstacles to discovering that they can actually do much more. To gauge atmosphere in a lesson, pay attention to the gestures and postures of learners *after* they have answered a question, as well as to the enthusiasm with which they volunteer.

"Guess what is in my mind" is not always initiated by the teacher. A learner who does not immediately know the answer to a teacher's question is very likely to start trying to 'guess around' the topic in the hope of stumbling on it. Tell-tale signs are a sequence of increasingly unthinking responses with a taste of 'guesswork'.

2.5 Being Genuinely Interested

The secret of effective questioning is to be genuinely interested not only in what learners are thinking, but in how they are thinking, in what connections they are making and not making. Genuine interest in the learners produces a positive effect on learners, for in addition to feeling that they are receiving genuine attention, you can escape the use of questions to control and disturb negatively. Instead of asking for answers, which in most cases you probably already know, you can genuinely enquire into their methods, their images, their ways of thinking. In the process, you demonstrate to learners what genuine enquiry is like, placing them in an atmosphere of enquiry which is, after all, one view of what schooling is really intended to be about.

If you are genuinely interested, you will wait for an answer when you have asked a question. Learners pick up quickly from the habit of asking and then answering your own question, or from a barrage of questions with little pause between them, that the questioner is not actually interested in an answer. Try holding yourself very still when you ask a question, and think about it yourself while waiting for an answer. If no response is forth coming, get them to talk to each other about the question for a few seconds, and then ask for contributions. Make it clear that every contribution is valued (e.g. record everything said on a board). Sometimes a smile with an eyebrow raised while looking (not staring or glowering) at a learner will encourage them to respond.

2.6 Teaching as a Caring Profession

Teaching is a caring profession: caring for both learners and mathematics, and it is maintaining a balance that can be difficult. It is all too easy to simplify questions and tasks so that everyone can succeed without being significantly challenged (learners quickly see through this strategy anyway) and equally easy to go over their heads with excessive challenge and sophistication. As Stein, Grover & Henningsen (1996) found, the most common action for teachers using prepared tasks was to simplify the task until it is obvious to learners what they need to do. This is the task analogue of funnelling, and it is equally unfulfilling for both teacher and learners. Maintaining the mathematical challenge was the single characteristic discerned in the first TIMMS videos (Stigler & Hiebert 1999) as distinguishing lessons from high-performing and low-performing countries.

2.7 Enculturating Learners Into Using Specific Questions Themselves

Make a list of the types of questions you would like learners to internalise, such as the following:

What do I know?	What do I want?
What do the words mean?	Can I state the question in my own words?
Can I depict the situation on a diagram?	Am I convinced?
Will it always work or happen?	Can I find an example?
Can I simplify the problem first?	What helped me get unstuck?

How is similar to or different from what I've done before?

Select a few and work on using those consistently over a period of time, then begin to use more and more indirect references to them. Establish a pattern of work in which learners ask each other for help when they are stuck, before asking you.

3. Attending to Attention

If there is something to the conjecture that questions disturb the flow of attention of learners, then effective use of questions would be built around gaining insight into what learners are attending to, and being aware oneself of how learner attention could most usefully be focused. To do this requires being aware of how your own attention is structured.

3.1 What is the learner attending to?

A teacher is demonstrating how to work through a particular problem. The teacher is aware that this problem is a particular case of a general class of problems, and sees it that way (seeing the particular in the general). The numbers are merely representative of any (relevant) numbers that could appear, while the structural constants are seen as common to all such problems. But the learners may only be aware of the particular problem being solved. They may be trying to work out what the rules are, what the steps are in solving it, without being aware of generality. They may not be ready to attend to and distinguish what is generic and what is particular in the resolution of the problem.

An important question therefore is what learners are attending to: what features are they stressing, and what are they consequently ignoring? Unfortunately asking such questions directly is rarely informative: learners usually don't know how to answer. However, there are some less direct ways of revealing something of what they are stressing.

One way is to get learners to read a problem or statement out loud, as they may reveal from voice tones and stress what is meaningful and what not. Another is to get them to 'say what they see' as they look at an expression, a diagram, a picture, a poster, a computer screen, etc.. No fancy technical terms are needed, just describing some aspect or feature. What they choose to describe is most informative, as long as you remember that absence of evidence is not evidence of absence: just because something is not mentioned does not mean that it is not seen, only that no-one has chosen to refer to it. Probably the most effective way to enquire into what learners are attending to is to get them to construct examples: examples of similar questions, examples of mathematical objects satisfying certain properties or conditions, and so on (Watson & Mason 1998), which is taken up in section 4.

3.2 What do I want the learner to attend to?

In order to use questioning effectively for focusing attention, it is necessary to do more than simply ask a question whenever an idea pops into your head. As a first step, it is necessary to become aware of what features you are stressing, and to make sure that your actions, both overt and covert, serve to stress or highlight those same features. By pausing after saying something significant, by pointing physically and verbally, by getting learners to try to say to each other something you have just said, you can assist them to focus on what you think is central and essential. You can also try out various forms of questions in an attempt to bring those features to the fore in learners' minds, however subtly or explicitly. This is the role of meta-questions mentioned earlier.

As a supplement to verbal questions, a useful strategy (known as variation theory) is to offer three or more examples in which the important features are varied, so that learners become intuitively aware of those features *as* features that can vary. Asking learners 'what is the same, and what different' about the examples is also helpful in drawing attention to what is mathematically significant (Brown & Coles 2000).

Attention can be multiply structured. Sometimes we gaze, taking in and holding a 'whole'. Even though in the background we are aware of details, it is the whole that holds the gaze. Sometimes the focus is on discerning details, locating parts. It is a natural use of human sense-making powers to try to recognise relationships between details we have discerned. of course, recognition and discernment often take place together or in quick succession. Sometimes we become aware that a relationship recognised in the particular situation is actually an instance of a more general property. Only when we are aware of properties is it possible to reason mathematically based on the use of agreed and acknowledged properties. All this is relevant when a teacher is talking to or with learners: if learners are gazing, they may not be able to hear or make sense of what is said about details they have not yet discerned; if the teacher is talking about relationships between as yet un-discerned or located details, then learners may not appreciate what is being said; if the teacher is thinking or speaking in terms of properties while the learners are concentrating on recognising particular relationships, then they may not grasp what is being said; if learners are focusing on the instantiation of properties, they may not be in a position to attend to reasoning based on those and other properties. Thus not only what learners are attending to, but how, is critical for working effectively with them as a teacher (Mason 2006, 2010).

4. Stimulating Learners to Question

How can learners be encouraged to take initiative, to be active learners rather than passive receivers? Simply asking a lot of questions is not enough. What matters is the form and type of questions asked.

4.1 Transforming Standard Questions

Watson & Mason (1998) collected a range of typical questions posed by mathematicians, and suggested ways of incorporating these in the classroom so as to provide learners with exposure to the breadth of typical mathematical questions. To see how some of these might be used, here are some ways of transforming the typical set of questions 'draw the graphs of the following … functions', in order to illustrate various types of questions that can be employed in many different mathematical topics.

Modified Question	*Underlying Principles*
Here are some graphs: suggest or pick out equations which could correspond to them.	Here is the answer to a typical question, what could the question have been (Doing and Undoing)?
Is it always, sometimes, or never true that two straight lines have one point in common? … that the equation of a straight line can be written in the form $y = mx + c$? … that a function which takes both positive and negative values must be zero somewhere?	Will something always, sometimes, or never be true?
What changes are needed to turn a quadratic with no real roots into one that does? What changes are needed to turn a straight line meeting a quadratic in two distinct points into one meeting it in two coincident points?	What changes are needed to turn a non-example into an example or *vice versa*?
What changes and what stays the same when you translate a straight line (in the graph, in the equation).	What changes and what stays the same when you …?
Express or describe the equations of all straight lines through a given point, or all quadratics tangent to a given line, …	Construct an example with the following properties …
What is the same about the equations of lines through a point? … quadratics which do not cross the x axis?	What is the same and what is different about …?
How is the axis of symmetry uncovered from the equation of a quadratic?	Is there another way to describe, depict or symbolise …?

Sort the following collection of graphs in some way, and write down a description of your classification principles; look at someone else's sorting, and decide on a classification system that will achieve that sorting (see for example Taverner 2000)	Sorting often reveals what a learner is thinking about when faced with the objects (examples, questions, …)

4.2 Learner Generated Questions

The action of getting learners to generate their own questions transforms the learners' relationship with authority and with tests (Holt 1968 p147) while at the same time giving them opportunity to exercise their creativity (even silliness) while still developing their mathematical thinking and their skilful use of techniques. Learners who have regularly and consistently made up their own questions similar to standard questions are more likely to be independent of their teacher, and in the position of recognising not only the type of question being asked (and hence having access to a method of approach), but also of having developed confidence in being able to tackle *all questions of a given type* not just the ones they have done for their revision.

Here are three sets of three questions each of a 'type'. For the third type, fill in the remaining boxes yourself to see what it might be like for learners.

	Type One	Type Two	Type Three
e.g. 1	$(x + 2)(x + 3) = x^2 + 5x + 6$	The sum of two numbers is 100 and their difference is 42, find the numbers	*A price is increased by 25%, but then put into a sale marked at 10% off. What is the actual markup overall?*
e.g. 2	$(x + 1)(x + 4) = x^2 + 5x + 4$	The sum of two numbers is 111 and their difference is 7, find the numbers	*A price is increased by 25%, then put into a sale. What is the reduction in the sale so that the actual price rise is only 10%.*
e.g. 3	$(x + 1)(x + 3) = x^2 + 4x + 3$	The sum of two numbers is 28 and their difference is 5, find the numbers	*A price is increased by 10% each year. In how many years will the price first be at least double the starting price?*

Really easy	$(x + 0)(x + 0) = ?$	The sum of two numbers is 10 and their difference is 0, find the numbers
Moderately difficult	$(x + 7)(x + 9) = ?$ $(x - 7)(x - 9) = ?$	The sum of two numbers is 137 and their difference is 43, find the numbers
Hard	$(x + 2.7)(x + 3.8) = ?$	The sum of two numbers is 19/17 and their difference is 3/5, find the numbers; The sum of two numbers is 3 and their difference is 15, find the numbers
General	$(x + \mathrm{a})(x + \mathrm{b}) = ?$ $(x + a)(x - b) = ?$	The sum of two numbers is S and their difference is D, find the numbers

Extension	$(ax + b)(cx + d) = ?$	*Twice one number and thrice another is 4, … find the numbers*

Note that what is seen as 'hard' starts to change as learners become aware of the generality of the class of similar questions and the generality of the technique for resolving them. What usually emerges from asking for a difficult question of a given type is that what makes it hard is the type of numbers involved, not the structure itself. Until learners have become familiar with being asked to make up their own questions and to experiment with constructing variants of different forms, they are likely to reproduce the format pretty closely. But once they have gained confidence, they will start to construct all sorts of extensions and variants, sometimes producing hard, even very hard questions.

The questions in the second column are based on the very first problem posed by Diophantos around 250 AD in his monumental work on solving equations (Heath 1964). Later authors put the same question into different contexts. You can alter the context, the numbers, or add to the features:

> A horse and its saddle cost £4.80, and the horse cost £1.20 more than the saddle. How much did each cost?
>
> A man rows 10 miles downstream in 2 hours and returns in 2 hours and 30 minutes. Find the rate of the river and his rate in calm still water. (Hawkes 1909 p215).
>
> The sum of three numbers is given, as are the differences between the first and the second, and the second and the third. Find the numbers. (Diophantus: Heath 1964)

More generally,

> Construct a question with the same numbers but different context that makes some sense.
>
> Construct a question with different numbers in the same context and same structure.
>
> Construct a question with different numbers and different context but same structure.

Construct a question with more features in addition to the ones in this problem but with the same basic structure.

5. Responding to Learners' Questions

Suppose a learner asks you how to do something, perhaps a fact or some technique they are supposed to know already. You have a choice: you can decide that it is more important that they make progress on the main topic (and so tell them directly), or you can decide that they need to refresh their skill and reconstruct it for themselves (and so make some suggestion or ask a pertinent question). Answering a question with a question may be attractive, but it can be excruciatingly irritating to a learner seeking information, as many teachers find when their own children get the 'teacher treatment' when seeking help with their homework: "Don't ask me a question, just tell me"! Establishing an overt contract with the learners is valuable in such cases, by finding out what sort of a response they are seeking and then providing it, but making an agreement to work on the issue later if need be.

However, what learners need is not for a teacher to resolve all their uncertainties, answer all their questions, or tell them what they do not remember. Rather what they need is to become familiar with *how* to deal with getting stuck: for example, looking for confidence inspiring examples to try out to see what is going on; looking up technical terms in order to check meaning and replace them with something more confidence inspiring; and clarifying what they actually know and what they need in order to solve the problem.

If a learner does not understand, they are most likely to ask for a repetition ("could you say that again please?", "could you go through that again please?"). If as teacher you always accede to this request, you train learners in dependency and you preserve your role of authority. You can choose instead, sometimes, to get someone else to 'say what they think you said', in order to stimulate learners to listen to each other and to learn from each other. It doesn't matter if someone doesn't repeat what you said, or even gets it twisted, because you can then all work on it together, while as teacher you become aware of some uncertainties in the class. If you can establish a practice in which learners are willing to struggle out loud because they know that others will help them (rather than mock or ignore them), then you and they will find that learning becomes more efficient as well as more satisfying.

6. Summary

Although a very common activity, question asking is at best problematic and at worst an intrusion into other people's thinking. By catching yourself expecting a particular response you can avoid being caught in a funnelling sequence of 'guess what is in my mind'. By being explicit at first, then increasingly indirect in your prompts, you can assist learners to internalise useful questions which they can use for themselves to help them engage in effective and productive mathematical thinking. Above all, the types of questions you ask will quickly inform your learners of what you expect of them, and covertly, of your enacted philosophy of teaching.

The key to effective questioning lies in rarely using norming and controlling questions, in using focusing questions sparingly and reflectively, and using genuine enquiry-questions as much as possible. This means being genuinely interested in the answers you receive as insight into learners' thinking, and it means choosing the form and format of questions in order to assist learners to internalise them for their own use (using meta-questions reflectively). The kinds of questions you ask learners indicates the scope and breadth of your concern for and interest in them, as well as the scope, aims, and purposes of mathematics and the types of questions that mathematics addresses. For more examples in secondary school, see Watson & Mason (1998), and for the same ideas in primary school, see Jeffcoat *et al.* (2004).

Bibilography

Ainley, J. 1987, Telling Questions, *Mathematics Teaching*, 118, p24-26.

Anderson J. & Boylan, M. 2000, The National Numeracy Strategy: teacher questions and pupil anxiety, in B. Jaworski (Ed.) *Proceedings of BSRLM* 20 (1 & 2) p49-54.

Baird, J. & Mitchell, I. 1986, *Improving The Quality of Teaching and Learning: an Australian case – the PEEL project*, Monash University, Melbourne.

Baird, J. & Northfield, F. 1992, *Learning from the Peel Experience*, Monash University, Melbourne.

Bauersfeld, H. 1995, "Language Games" in the mathematics classroom: Their function and their effects. In P. Cobb & H. Bauersfeld (Eds.), *The Emergence of Mathematical Meaning: interaction in classroom cultures*, Lawrence Erlbaum Associates: Hillsdale, NJ p271-291.

Brown, L. & Coles, A. 2000, Same/Different: a 'natural' way of learning mathematics. In T. Nakahara and M. Koyama (Eds.) *Proceedings of the 24th Conference of the International Group for the Psychology of Mathematics Education*, p2-153-2-160, Hiroshima, Japan.

Brown, M. & Kuchemann D. 1976, 'Is it an add, miss?', *Mathematics in Schools*, 5(5) p15-17.

Brown, S., Collins A., & Duguid P. 1989, Situated Cognition and the Culture of Learning, *Educational Researcher* 18 (1) pp. 32-41.

Bruner, J . 1986, *Actual Minds, Possible Worlds*, Harvard University Press, Cambridge.

Hawkes, H. 1909, *Complete School Algebra*, Ginn & Co. Boston.

Heath, Sir T. 1964, *Diophantus of Alexandria: a study of the history of Greek algebra*, Dover, New York.

Holt, J. 1964, *How Children Fail*, Pitman, London.

Holt, J. 1968, *How Children Learn*, Pitman, London.

Jeffcoat, M. Jones, M. Mansergh, J. Mason, J. Sewell, H. & Watson, A. 2004, *Primary Questions and Prompts*, Association of Teachers of Mathematics, Derby.

Love, E. & Mason, J. 1992, *Teaching Mathematics: Action and Awareness*, Open University, Milton Keynes.

Mason, J. 2002, Minding Your Qs and Rs: effective questioning and responding in the mathematics classroom, in L. Haggerty (Ed.) *Aspects of Teaching Secondary Mathematics: perspectives on practice*, RoutledgeFalmer, London, p248-258.

Mason, J. 2006, Micro-Structure of Attention in the Teaching and Learning of Mathematics. *Proceedings of the Mathematics Teaching 2005 Conference*, Edinburgh: Edinburgh Centre for Mathematical Education. p36-41.

Mason, J. 2010, Attention and Intention in Learning About Teaching Through Teaching. In R. Leikin & R. Zazkis (Eds.) *Learning Through Teaching Mathematics: development of teachers' knowledge and expertise in practice*. p23-47. Springer, New York.

Stein, M. Grover, B. & Henningsen, M. 1996, Building student capacity for mathematical thinking and reasoning: An analysis of mathematical tasks used in reform classrooms. *American Educational Research Journal*, 33(2), 455-488.

Stigler J. & Hiebert, J. 1999, *The Teaching Gap: best ideas from the world's teachers for improving education in the classroom*,Free Press, New York.

Taverner, S. 2000, Challenging 11-14 Year Olds, *Mathematics in School*, 29 (3), p17-20.

Watson, A. & Mason, J. 1998, *Questions and Prompts for Mathematical Thinking*, ATM, Derby.

Wood, T. 1998, Funneling or focusing? Alternative patterns of communication in mathematics class. In H. Steinbring, M. G. Bartolini-Bussi, A. Sierpinska (Eds.), *Language and Communication in the Mathematics Classroom* (pp. 167-178). Reston,VA: National Council of Teachers of Mathematics.

Wood, D., Bruner, J., & Ross, G. 1976, The Role of Tutoring in Problem Solving. *Journal of Child Psychology and Psychiatry*, 17, p89-100.