2

Do resources matter in primary mathematics teaching and learning?

Doreen Drews

It has been common practice for many years for primary school teachers and Foundation Stage practitioners to view the use of resources as an essential part of teaching and learning mathematics. Foundation Stage and lower primary teachers, in particular, have drawn upon aspects of constructivism to validate the argument that children operating in the Piagetian stages of 'pre-operational' and 'concrete operational' modes of thinking need to manipulate objects to make sense of, and develop, mathematical ideas. While acknowledging that the teaching and learning of mathematics does benefit from effective use of visual and practical aids, recent research has questioned whether such use is always needed, or helpful, to children's mathematical understanding. Crucial to the debate is the rationale which teachers use to support the planned use of mathematical resources within their lessons (Moyer, 2001), teacher beliefs about how best to teach mathematics to assist children's learning (Askew *et al.*, 1997), and assumptions which teachers may make regarding children's interpretations of the use of mathematical resources (Cobb *et al.*, 1992).

Chapter focus

This chapter will explore the role that a wide range of resources could play in effective mathematics teaching and learning in the primary years. Current research will be reviewed to support, and challenge, the contention that the manipulation of practical aids is helpful, and often necessary, to the development of children's mental images of mathematical concepts. It will ask the question whether or not practical work and the use of iconic imagery directly link to symbolic understanding. It will focus on:

- **the value of resources to the teaching and learning of primary mathematics;**
- **the issues involved in the use of resources to support mathematics teaching and learning;**
- **the critical aspects involved in the choice and use of resources to support effective mathematics teaching and learning.**

The value of resources to the teaching and learning of primary mathematics

A rationale for using mathematical resources

In the area of mathematical learning, Jerome Bruner's three modes of representing our experiences (1964) are considered important to the development of children's understanding: the enactive mode involves representation of ideas through undertaking some form of action

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(such as manipulating physical objects); the iconic mode involves representing those ideas using pictures or images; and the symbolic mode involves ideas represented through utilising language or symbols. By the 'representation of ideas', Bruner was referring to the outcomes of the processing of past experiences. For Bruner (1966), these modes of representation are mutually supportive in assisting the storage of 'pictures in the mind'. This entails the development of a mental 'storage system' which allows learners to make predictions and to retrieve relevant information from past experiences to extrapolate to new situations. The use of physical resources, models and images in mathematics teaching and learning relate well to the enactive and iconic modes of representation, with mental imagery and language supporting the understanding and use of symbols.

The interconnections between manipulation of objects, iconic imagery, use of language and symbols can, perhaps, be more commonly seen in activities involving young children, although Edwards (1998: 18) argues that mathematical understanding is brought about for all children by connections being made between these modes of representation. Haylock and Cockburn (2003) suggest that the network of connections between concrete experiences, pictures, language and symbols could be significant to the understanding of a mathematical concept (Figure 2.1).

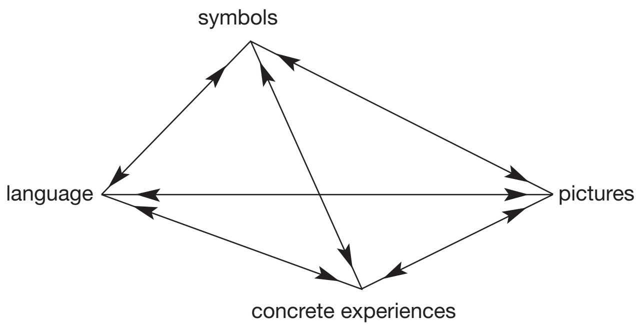

Figure 2.1: Significant connections in understanding mathematics

Central to this is the notion that 'when children are engaged in mathematical activity … they are involved in *manipulating* some, or all, of the following: concrete materials, symbols, language and pictures' (Haylock and Cockburn, 2003: 3). It is this act of manipulation that allows for connections to be made through the different experiences. Moyer (2001: 176) supports this by stating that it is the active manipulation of materials that 'allows learners to develop a repertoire of images that can be used in the mental manipulation of abstract concepts'.

It would seem, therefore, that a key aspect to the value of children using practical resources within mathematics is a need for such activity to have a role to play in the development of mental imagery and mental strategies. Beyond this, a review of research undertaken by Askew and Wiliam (1995: 10) showed that 'practical work can provide images that help pupils contextualise mathematical ideas. It can also provide experiences out of which pupils can abstract mathematics.' In order for this to happen, Delaney (2001: 128) suggests that mental imagining of the given resource, and any action undertaken with the resource, need to be 'internalised and used to process mathematics when the resource is not physically present'. This would suggest that an important part of a teacher's role is to plan activities and conversations which refer to previous experiences of practical activity and encourage children to develop mental images.

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Chapter 3 develops this idea further with specific case studies focusing on children building, and using, internal resources.

While concurring that practical activity has a clear role in aiding children's mathematical development, Anghileri (2000: 10) cautions against an overuse of concrete materials: 'It is important that children do not come to rely on using such materials for modelling numbers but that they develop mental imagery associated with these materials and can then work with "imagined" situations.' Hughes (1986) showed that young children were capable of imagining objects within a 'box game': this appears to be an important stage between manipulating objects and abstract work.

Askew (1998: 8) agrees that practical work on its own is not enough and suggests that practical activity should have an element of 'in the head' to avoid giving young children, in particular, the impression that mathematics is only about practical work.

The selection and effective use of appropriate mathematical resources requires careful consideration and planning on the part of the teacher. As Bottle (2005: 84) points out, the appropriateness of the resource should be 'judged by the extent to which the mental images that children form as a result are likely to be helpful or unhelpful in structuring their thinking'.

Types of mathematical resources and how they may be used effectively

The term 'mathematical resource' is defined here as any form of specific mathematical apparatus (structured or unstructured), image, ICT, game, tool, paper, or everyday material which could be utilised to provide a mathematical teaching or learning aid.

Manipulatives

Specific mathematical apparatus, or manipulatives, are 'objects designed to represent explicitly and concretely mathematical ideas that are abstract.' (Moyer, 2001: 176). They can be used as models by both teachers and learners, hold a visual and tactile appeal, and, as such, are designed primarily for hands-on manipulation.

Structured mathematical apparatus is specifically designed to embody one particular conceptual structure. Through manipulation of the apparatus the learners, or teachers, 'directly reflect the equivalent mathematical manipulations within that structure'. (Bottle, 2005: 87). Examples of this type of apparatus are Multibase 10 (Diene's) material and Cuisenaire Number Rods. Both types of material reflect the relationships within our base 10 number system: for example, Multibase 10 can be used to model the base 10 place value system and the relationships between, e.g. one hundred and ten tens. This structure is particularly helpful in helping children make sense of decomposition as a strategy for subtraction where it needs to be understood that i.e. four tens and three ones is worth the same as three tens and 13 ones.

Cuisenaire Rods enable children to explore the properties of numbers and relationships between numbers. Number pairs, inverse operations and number patterns can be explored and familiarity with/use of the apparatus can assist mathematical understanding (Delaney, 1992).

Unstructured mathematical apparatus is often more versatile and 'open' in its use as it has not been designed to focus on particular conceptual structures. Examples include Multilink,

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many counting materials or collections of shapes. In all these examples the materials can be used in specific ways, i.e. to aid counting, or in more exploratory ways such as investigating number/shape patterns, construction and design, or relationships between types of numbers or shapes. Variety of the same type of resource is important in the selection in order that children do not form misconceptions based on experiences with limited resources. As Hansen (2005: 85) indicates, where children are only exposed to prototypical examples/images of shapes, they are likely to form incorrect generalisations. The ability to use these materials in diverse ways can promote greater opportunities for investigational and collaborative work: such activities are more likely to encourage purposeful mathematical discussion and development of logic and reasoning. (Examples can be found in the case studies in Chapter 4.)

Images

Young children will link an acoustic image (sounds, rhythms) to a concrete image of something meaningful such as fingers or objects (Harries and Spooner, 2000: 49). Visual and tactile images, such as an abacus or bead string, assist children in linking counting to movement. Such resources help children develop a sense of number order and number pattern, particularly where the beads are blocked in groups of fives or tens as evident on bead strings or a Slavonic abacus. Through the use of colour and/or groupings of beads, the concepts involved are embedded within the image. In order to connect this sense of order with symbolic representations of number a more abstract image is needed.

An abstract image gives an opportunity for particular aspects of mathematics to be presented as either visual or 'hands-on' teaching and/or learning aids. These could include pictorial images of shapes/objects or images more closely connected to our symbolic number system, e.g. number tracks, number lines, digit cards, 100 squares and place value arrow cards. As Delaney (2001: 132) points out, the latter abstract representations appear to be favoured by the National Numeracy Strategy (NNS) (DfEE, 1999) over more 'concrete' specific mathematical apparatus without a clear rationale evident for the choice. The value of connecting concrete images (as described above) with abstract images is that children learn to relate the number symbols, and order, to the acoustic and concrete images that they have experienced. (Examples of this can be found in case studies in Chapters 3, 5 and 6.)

As children develop their understanding of the number system, progression in the level of sophistication of an abstract number line is needed. More powerful images that illustrate how numbers are related in a logical structure act as a model for both teaching and learning. Consequently, number tracks need to be superseded by calibrated number lines as the latter allow for negative integers and intermediate values to be represented. A number line can therefore be a helpful teaching and learning aid beyond counting on and counting back. One aspect of fractions and decimals which children find difficult is the notion that they are numbers which fit within our number system (Frobisher *et al.*, 1999). As Lawton (2005: 40) points out, this lack of understanding is more evident when dealing with fractional/decimal values greater than one. Activities which demonstrate and which allow for rational numbers to be positioned on a number line can support a greater understanding of place value, relationships between 'numbers of a different kind' and our overall number system: this is true for all children including the more able in mathematics. An empty number line allows for the modelling of mental calculations where the order of numbers remains true but numbers and intervals are not marked. Children do not need to work to a correct scale in their size of 'jumps' between numbers. Harries and Spooner (2000: 50–51) view empty number lines, in particular, as providing children with flexible thinking tools: decisions need to be made at the point of construction regarding which numbers to use, how to place them and the intermediate numbers to be shown.

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Abstract images, therefore, provide reference tools and thinking tools for children to work with as they develop understanding of the number system and perform calculations with numbers. Such images can act as visual cues/memory aids which, in turn, act as a basis for reflective thought. In addition, models, images and diagrams should assist understanding of how a particular strategy or method was used to solve a problem and why it worked. Research undertaken by Clausen-May (2005: 84) shows that visual and kinaesthetic thinkers are more likely to benefit from teaching and learning approaches which make effective use of models and images 'that make key mathematical concepts manifest'.

ICT

A wide range of information and communication technology (ICT) is available in most primary schools as useful resources and tools to support the teaching and learning of mathematics. This can include programmable robots, calculators, television, radio, audio tape, video, digital cameras as well as computers, software, access to the internet and interactive whiteboards (IWBs). The NNS (DfEE, 1999: 32) advocates the use of such resources providing that 'it is the most efficient and effective way to meet your lesson objectives'. Bottle (2005: 95) lists some appropriate uses of ICT in mathematics lessons and suggests that there need to be connections between the tasks/activities undertaken using the ICT device and mathematical activities independent of the device. This is supported by Higgins and Muijs (1999: 112), who advocate that more explicit links need to be made between computer activities and other planned activities in order that pupils develop a greater awareness of mathematical connections. Anghileri (2001: 186) suggests that calculators are at their most effective as cognitive tools when they are used to provoke thinking rather than as simple machines to obtain answers to given calculations.

While there is little doubt that ICT offers powerful visual images and that children are motivated by ICT devices, on its own this will not necessarily lead to increased understanding of any mathematical aspect or concept. As with the use of all resources or tools, it is the choice of task, effective use of the resource/software, quality of teacher intervention and opportunities for discussion which are fundamental to successful learning. OFSTED (2005) continue to report that too few teachers use ICT effectively in their mathematics lessons. There does appear to be, however, a greater use of IWBs within whole-class lessons or sections of lessons, particularly for demonstration and review purposes. Chapter 8 provides more focus on creative and effective use of IWBs to develop mathematical thinking.

Mathematical games

Mathematical games can be played in whole-class, small group or paired settings. They are a resource which is usually highly motivating to children and, consequently, encourages greater levels of concentration and engagement with mathematics. Games can be used in different ways to consolidate learning, practise skills, explore mathematical relationships and develop problem-solving strategies. Many board games, commercially produced games and some computer games are designed to practise particular aspects of mathematics; Parr (1994: 29) sees this as a major advantage of games as they 'can stimulate people to give repeated practice to skills of mental arithmetic and then do the whole thing again simply because they want to do better the second time around'. While such games do allow for the use and application of skills in a different context, when choosing these types of games, teachers and practitioners need to give consideration to the mathematical content and the level at which the children are working. The best games will allow for different levels of challenge.

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One enjoyable aspect of games for children is that they are put into situations where they can control their own learning: there is often no 'one way' to solve the problem or achieve a winning solution. As Hatch (1998) and Anghileri (2000) note, this control encourages flexibility of thinking and mental fluency. The more effective games encourage mental work as calculations are tackled in children's heads. As much as possible, children should be encouraged to discuss the mathematics inherent in the game, and the strategies employed, in order to help the development of mathematical language and reasoning skills.

While many mathematical games are designed as competitive games, they can often turn into co-operative games where pupils support each other to obtain the greatest success. Carefully planned, these types of games can provide opportunities for developing skills related to mathematical thinking – predicting, generalising, justifying and explaining. (Chapter 4 has examples of such types of games in whole-class settings.) For Ainley (1988: 243), the main value of mathematical games lies in the linking together of mathematical problems which are 'real' to the children, the use of such process skills as listed above, and the need such activities present for children to think in a mathematical way.

While there are benefits in mathematical games being used as homework activities or 'standalone' free-choice activities in the classroom, the most effective use of games is when they are incorporated into the planned mathematics curriculum. Teachers and practitioners need to be clear on the intended learning outcomes of the game, how all children can benefit from appropriate games (not just the 'fast finishers'), and plan opportunities for adult support, discussion and pupil explanations.

Worksheets and textbooks

These feature strongly as mathematical resources in many Key Stage 1 and Key Stage 2 classrooms. As Harries and Spooner (2000: 46) point out, worksheets and textbooks play an important role in influencing teachers' thinking with regard to the teaching and learning of primary mathematics. Many commercial schemes exist, usually comprising of a teacher's guide (usually the most useful, but, ironically, often the most underused part of the scheme), children's textbooks/workbooks, and additional resources such as photocopy masters for worksheets/CD-ROM materials. As Liebeck notes (1984: 16), such resources focus primarily on pictures and symbols rather than on 'concrete' experiences and language. This is problematic for Atkinson (1992: 13), who views meaningful mathematics as 'maths with reason [which] is rooted in action – learning through doing'. She suggests that schemes of work, therefore, need to start off with activity.

A teaching/learning approach that relies on a predominant use of textbooks and worksheets for mathematics can produce difficulties for all children, not just young children:

- children with visual and kinaesthetic learning styles often struggle with a 'print-based curriculum' (Clausen-May, 2005);
- there are syntactic and semantic levels of reading and interpretation of the illustrations involved in textbooks which can lead to confusion (Santos-Bernard, 1997, cited in Harries and Spooner, 2000);
- a predominant use of worksheets can 'persuade' children that mathematics has nothing to do with the real world but, perversely, encourage an attitude that 'real' mathematics is textbook/worksheet work;
- work from worksheets and textbooks does not always reflect an accurate view of what children can do.

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In addition, problems can lie in the way in which textbooks and worksheets are used as activities/tasks. OFSTED (2005: para 63) notes that less effective teaching relies too heavily on worksheets, with children sometimes struggling to interpret the sheet/text or sustain interest in the work. An overuse of texts/worksheets also encourages teachers to view independent work as children working on their own, rather than simply independent of the teacher.

A more effective approach to the use of children's textbooks and worksheets is to view them as resources which may be useful to support, consolidate or extend children's mathematical learning through linking selected aspects to the unit of work planned by the teacher. Such an approach allows for teachers to make decisions on the appropriateness of the material, which groups of children may benefit from the set task, and to plan for independent work that is paired based/group based focusing on explanation of understanding. In these ways textbook activities, in particular, are used as a springboard into further problem-solving/investigational tasks with the benefit of children making stronger connections with 'textbook maths', other forms of mathematical activity, the use of mathematical thinking skills and doing 'real' mathematics.

Everyday materials

These can be brought into the classroom and used successfully as resources to support and develop children's understanding of some of the purposes of mathematics in real-life contexts. The examples of materials which help relate 'school' mathematics to everyday applications are endless, but could include packaging materials, patterned fabric or paper, timetables, receipts, catalogues, scaled plans, photographs of shape/number in the environment and any form of container or measuring device. Such types of resources have use in whole-class teaching, small group activities, displays and cross-curricular role-play situations. The value of resources in role-play and other cross-curricular activities is explored further in Chapter 6.

In addition to these real-world artefacts, many resources not specifically designed for mathematical learning can be exploited to assist with early learning in particular. Toys, stories, environmental or malleable materials such as sand, water and play-dough can be used to support early concepts in aspects of number, shape and measurement. The advantages here are that they are tactile and more likely to connect with children's home/prior/real-world experiences. For Edwards (1998: 8), the value here lies in the fact that 'handling of familiar "everyday" objects enables children to learn about their properties and components'. Through manipulating familiar objects and materials, children are helped to rationalise their experiences. Aubrey (1997: 26) sounds a word of caution, however. Her research indicates that young children do not often relate their classroom interactions with the associated use of materials to their existing out-of-school problem-solving. In children's real-world experiences objects and materials are used in problem-solving situations, often play-based, and often self-initiated. This suggests that the use of everyday (and specific mathematical) resources is more successful in supporting children's learning through the type of teaching approach, and classroom environments, which put high priority on solving problems which are meaningful to the children.

Through reviewing a wide range of resources, it is possible to identify their potential to:

- motivate children;
- provide variety to teaching and learning experiences;
- connect 'classroom mathematics' with application to the real world;
- act as a visual aid to allow children to build up a store of mental images;

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- enable teachers and children to model mathematical processes involved in specific number operations or calculations;
- encourage mathematical communication to take place;
- support teacher assessment of children's knowledge and understanding of aspects of mathematics;
- support the understanding of mathematical ideas through allowing children to make connections between, what for them may be, disconnected aspects of mathematical learning.

Anghileri (1995: 7) argues that it is 'active participation in problem solving through practical tasks, pattern seeking and sharing understanding' that enables children to make their own sense of the relationships that underlie all mathematical knowledge. Crucial to this argument is the belief that effective use of resources provides a forum for the acquisition and use of mathematical language and purposeful discussions. Anghileri (2000: 8) believes that 'mathematical understanding involves progression from practical experiences to talking about these experiences, first using informal language and then more formal language ... Talking about their experiences will help children establish the significances of the vocabulary used and how they relate it to the visual imagery being created.' This process takes time. It is important to remember that 'there is no mathematics actually in a resource' (Delaney, 2001: 124). Abstraction is needed from all these experiences. This signals a clear need for teachers and practitioners to:

- be clear as to why they provide specific resources;
- recognise the links between the practical task, the visual imagery created and the abstract mathematics involved;
- view the use of such resources in part as a social activity which can assist with reflective discourse between children and adults;
- give high priority to questioning and discussion linked to how the children used or worked with the resource to support their mathematical thinking.

The issues involved in the use of resources

This section will focus on the necessity for children to abstract the mental mathematics from their practical experiences and the research undertaken in this area.

Hart *et al.* (1989) investigated children's ability to make the transition from practical work and pictorial images to more abstract mathematics. Many of the 11- to 12-year-old children in the project had difficulty in moving 'from the concrete or pictorial representations to the more formal (general) aspects of mathematics' (Hart *et al.*, 1989: 218). The research showed that many children were unable to link these stages in the learning process. Cobb *et al.* (1992) suggested that some of these difficulties derive from the use of particular materials which are used within a 'representational' approach. In this approach children would work with an external representation (e.g. Multibase10) in order to give meaning through 'internal' representations to a particular aspect of mathematics (in this example, aspects of place value). Cobb *et al.* suggest that there is an assumption here, on the part of the teacher, that specific mathematical meaning is actually embodied in the external representation: this may be true for the teacher, but not necessarily the child. Gravemeijer (1997: 316) concurs that 'concrete embodiments do not convey mathematical concepts' and that it is the 'experts' who already have those concepts who will make sense of the ideas being modelled.

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How materials are used, therefore, and the ability of the teacher to negotiate their differing interpretations, would appear to be important factors in helping children translate their thinking processes from handling objects/using images to symbolic representations. Children need to see through the objects to the mathematics which underpin the representation (Harries and Spooner, 2000: 46). They need to be able to think with the representations.

For Ball (1992), too many teachers believe that children will reach 'correct' mathematical conclusions simply by manipulating resources. As she points out, 'although kinaesthetic experience can enhance perception and thinking, understanding does not travel through the fingertips and up the arm' (Ball, 1992: 47).

In itself, therefore, the physical exploration of concrete materials will not lead to children 'discovering' mathematical concepts. For MacLellan (1997), the crucial element is accompanying mental activity. Without some accompanying mental activity to reflect on the purpose and/or significance of the physical activity, concrete materials will not actually enable the child's mathematical understanding to develop (MacLellan, 1997: 33). In order for this to happen, there needs to be a discourse between the child and the teacher/practitioner which will allow the child to bridge the gap between the materials and the abstract ideas.

The need for teachers to be aware of the different meanings that children can ascribe to the same resource is highlighted by research undertaken by Ahmed *et al.* (2004). This study suggested that 'different children will engage with the same materials in different ways depending on the conceptions they bring with them and, hence, will establish different understanding'. The same research indicated a lack of clarity in many teachers' thinking regarding the 'subtle distinction between the way mathematical ideas are constructed from objects and the particular characteristic of the objects' (Ahmed *et al.*, 2004: 320). This would suggest a clear need for children to describe the different ways in which they perceive the material and its relationship to the mathematical idea under discussion.

Threlfall (1996) examined the reasons why the theoretical benefits of practical activity in mathematics did not always translate into practice. The use of specifically structured apparatus as an aid to finding the answers to calculations was highlighted as unhelpful to children's understanding. 'If children with little awareness of number patterns or the structure of our place value system, who do not have much idea about the meaning of the arithmetic operation, are being taught how to do "sums", the use of the apparatus to demonstrate the procedures will not make any difference to the success of the task' (Threlfall, 1996: 7). For Threlfall, using structured apparatus in such situations only obscured the real value of the resource; namely, to provide 'contexts in which meanings can be established and extended, in which relationships can be exemplified and explored and in which techniques can be demonstrated' (ibid.: 11). The contention here is that, having had sufficient exploration of possibilities in number through engaging with the apparatus in suitable contexts, children should be able to work on the calculations without the use of the apparatus. If they cannot, then the children should not yet have been expected to work on such calculations. More effective exploration and use of the apparatus would help them to succeed on similar calculations at a later date.

The need for children to develop and use calculation strategies eventually without the aid of concrete 'manipulatives' was identified in research undertaken by Carpenter and Moser (1982). They found some evidence that young children who could mentally apply sophisticated counting or calculation strategies reverted to more 'primitive' counting or calculation strategies

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when materials were available. In such situations it could be argued that the availability of concrete resources is inappropriate as they act to 'slow down' the children's thinking processes. This would suggest that teachers need to give careful consideration not only to the types of resources to have available, but also as to whether they should always be available to all children. A problematic issue arising out of this is the impact on children's self-esteem and the desire of most children, particularly in Key Stage 2, to be 'seen' to be undertaking the same tasks as their peers in the class. In such situations, children who would benefit from using appropriate resources may be unwilling to use them even if they are available.

Moyer (2001) found evidence that some teachers did not choose to provide resources to aid the children's mathematical learning (even when they acknowledged they might be helpful) as such materials were deemed babyish for older pupils. The research highlighted that teacher decisions on using, or not using, mathematical resources stemmed from their inherent views on why they teach mathematics, and how it can be learned effectively. These views influenced their teaching approaches. For many of the teachers in this study, resources were used to simply add 'fun' into lessons rather than using them to relate to the mathematical ideas being explored. Through observations, questionnaires and interviews, it was concluded that the underpinning issue here was a lack of understanding on the part of these teachers as to how to represent mathematical concepts. Without this understanding, the resources became used as little more than a diversion.

The importance of teacher beliefs into the 'why' and 'how' of teaching mathematics was clearly demonstrated by research into effective teachers of numeracy undertaken by Askew *et al*. in 1997. One of the characteristics of the most effective teachers in this study (deemed to be 'connectionist' teachers) was their ability to use and move between a wide range of different representations of mathematics – concrete objects, images, language and symbols – and to make connections being these different representations for their pupils.

The critical aspects involved in the choice and use of resources

It is clear from the previous section that 'practical work is not at all useful if the children fail to abstract the mental mathematics from the experience' (Askew, 1998: 15), and that teachers need to be clear as to the purpose of using specific resources. This section will explore the critical aspects involved in teacher decision-making with regard to making the most effective use of mathematical resources to support teaching and learning.

Moyer's research alerts all practitioners that choosing to use resources in mathematics education on the basis that they provide more 'fun' for children is a simplistic approach at best and, at worst, can lead to situations in which both teacher and pupil are confused as to how such resources are beneficial to mathematical learning (Moyer, 2001).

For Delaney (2003), part of the decision process revolves around being clear as to whether resources are more effective when used in teacher demonstrations or used by children to engage with mathematical ideas. The latter is advocated as giving children a feeling of personal involvement and providing greater scope for the development of skills related to mathematical thinking. The disadvantage of an over-reliance on a 'demonstration' approach is that 'if a resource is only ever used to demonstrate how to do something you will only know from the child's actions whether they understood the *instructions* given' (Delaney, 2003: 41).

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In addition, when selecting resources, teachers need to be secure in the purpose behind using a particular resource and being clear as to the support it can offer children. For example, place value arrow cards are useful to emphasise how number names are written and how the number value can be represented in hundreds, tens and ones, but they cannot offer a concrete sense of the size of the number. (An example of this can be found in one of the case studies in Chapter 3.)

Diversity is important in choosing resources to support children's learning. Children can form incorrect generalisations if they are only presented with limited examples. It is therefore important that children are given, and talk about, examples and non-examples so that they can investigate relevant and irrelevant features (Askew and Wiliam, 1995).

A greater use of mathematical resources in open-ended tasks may encourage teachers and children alike to view objects/images as tools or representations to help thinking. Flexible uses of resources can encourage flexible thinking. This approach can help develop a classroom 'culture' in which it is recognised that there are many paths to reach the same mathematical solution. In turn, some of those paths may involve the use of resources for some children. Turner and McCullouch (2004: 65) suggest that allowing choice in resources (either from a wide range or a selection chosen by their teacher) 'enhances the ability of children to apply their knowledge to new situations'. Choice may often depend on the child's preferred learning style.

Above all, actions as well as the intentions of the teacher are important when using resources as teaching or learning aids. Whatever the material provided or the context chosen, assumptions should not be made that children will draw the mathematical conclusions from the resource simply by interacting with it. The use of any form of mathematical resource (as defined in this chapter) needs to be accompanied by child–child and child–adult dialogue in order to:

- diagnose any misconceptions perhaps more evident through use of the resource;
- establish the level of mathematical understanding;
- use and apply relevant mathematical vocabulary;
- assess the effectiveness of the resource as an aid to learning and as a mechanism to support the development of mathematical thinking.

SUMMARY OF KEY POINTS

Resources have an important role to play in allowing teachers to model or demonstrate representations of mathematical ideas, and in supporting children's developing mathematical understanding and thinking. The effective use of any resource will depend on teacher understanding of how the particular representation helps develop mental imagery, and how to utilise the resource to assist with children's understanding of particular mathematical concepts. The process of abstracting mathematical ideas from practical aids or images is difficult for many children: all teachers and practitioners need to be mindful that 'just because the child is presented with some concrete materials it does not follow that the child will abstract the mathematical ideas from the materials' (MacLellan, 1997: 33). Significant to assisting this process of abstraction appears to be choices made by teachers regarding the type of resource to be used, the role which teachers see themselves as having while children are engaging with the resource(s), and the social culture of the class.

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