Students'
Worksheets

FRACTIONS AND PROBLEM SOLVING

Session 1

Activity 1.1

PLAYING CARDS

This is an activity to practice:

- New vocabulary: Elements of fractions, types and procedures.
- Conversion from one expression of a rational number to another.

INSTRUCTIONS

- Work in pairs
- Each pair of students has a set of 16 pairs of cards representing the same rational number but expressed in a different form For example: 25% and 1/4 split up into two piles.
- One pile has to be spread on the table face up.
- The other cards are in a pile, face down
- In turns, take a card, read or describe. Your partner has to understand and find the matching card on the table. If the student is wrong, the card described is placed at the bottom of the pile, but If is right, he keeps the matched pair. Students swap roles.
- The game finishes when there aren't any more cards in the pile
- Correct pairs are shown on the board.

You can work out the answers on paper.
Don't use calculator.
English only!

Session 1

Activity 1.2

QUESTIONS

This is an activity to:

- Use language to justify your answers and to convince your partner you are right, if there is disagreement.
- Consolidate conversion procedures.

INSTRUCTIONS 1

- Work individually
- Choose an answer to the following questions, working them out if necessary. Be ready to justify your answer using the "speaking frames" for "Giving reasons" projected on the board.
- Check the answers with your partner
- Justify your answers, especially if you disagree.

INSTRUCTIONS 2

- Work in pairs
- Write a new test, based on the pictures. Follow the example of question paper 1.
- Share your new questions with peers in plenary. Be ready to justify options taken.

You can work out solutions on paper.
Don't use a calculator.
Be aware with British notation for mixed numbers!
English only!

1. Question paper 1:

a) $^3/_4$ is the same as: ○ 0.75% ○ 34% ○ 0.75	**b) $^1/_8$ is the same as:** ○ 0.125 ○ 1.25 ○ 8%
c) $^2/_3$ is the same as: ○ 23% ○ $33^1/_3$% ○ $66^2/_3$%	**d) 5. 2% is the same as:** ○ 0.52 ○ 5/2 ○ 2.00
e) $^1/_4$ is equivalent to: ○ 0.4 ○ 25% ○ 40%	**a) 0.3 is equivalent to:** ○ $^1/_3$ ○ $^3/_{10}$ ○ 33.3...%

2. Question Paper 2:

Write a Multiple Choice question to each picture

	1. Josep sees that to Dunnotar Castle it is _____ miles. It is the same as ○ ... ○ ... ○ ...
	2. Joan measured the thickness of ice: It's 0.433..."(inches).
	3.
	4.

Session 2

Activity 2.1

WORD PROBLEMS

This is an activity to:

- Practice operators: Find the part, the quantity or the fraction.
- Use language to read and understand short problems about operators in different contexts.
- Use language to summarize instructions of short problems orally and to explain a simple procedure.

INSTRUCTIONS 1

- Work in pairs
- There are 10 questions to solve in 15 minutes.
- In turns, one student reads carefully one word problem and summarizes it to the partner.
- You must solve the problem together and write the working out and answer on a paper.
- Swap roles
- Check answers and procedures in plenary

INSTRUCTIONS 2

- Work in pairs
- There are five answers without instructions. Write the problem which could match each answer. It must be a real problem, not an abstract one. Be clear and use your imagination.
- Finally, create five new problems freely, using different contexts, as you have seen in the examples.
- Swap the questions you have created with another pair. Do the questions as homework.

1. Questions to solve

Instructions	Working out & solution
1. Two hundred people take part in a survey. Three quarters say that they like to go to the cinema. How many people is this?	
2. When a box of eggs is dropped, 2/3 of the eggs are broken. If the box holds 18 eggs altogether, how many are not damaged?	
3. To pass an exam you need 75% of the 60 marks. How many is that? If you have got 54 marks, what percentage is this? What fraction?	
4. In a school, 7/12 of the pupils are boys. There are 384 pupils in the school. How many girls are there in the school?	
5. ¾ of houses in a street have satellite TV. If there are 5 houses that do not have satellite TV, how many houses are in this street?	
6. In a school, 153 pupils have pets. There are 510 pupils in the school. What fraction of pupils has pets? What percentage?	

7. In a class there are 32 pupils. Of these, 3 /8 come to school by bus, ¼ by car, 3/16 by bicycle and the rest on foot. How many pupils come to school by bus, by car, by bicycle or on foot?	
8. A Numeracy class records the weather during April. Work out the fraction and percentage of days in each case. (April=30days) **- 3 days below freezing** **- 6 days warm and sunny** **- 9 days wet** **-12 days cloudy** **-18 days changeable weather**	
9. You are going on forty-mile trip, and you have already covered two thirds of the way. How many Km are you away from your destination? (More or less, 1 mille = 1.6 Km)	
10. Lara is training for the Boston Marathon. **Yesterday she ran 7/8 of the total course in 3 hours. Today Lara ran at the same speed but was able to finish the entire course. How long did it take Lara to complete it? Give the answer in minutes.**	

2. Given the answer, write instructions

Instructions	Working out & solution
	$\frac{2}{5}$ of $40 = \frac{2}{5} \cdot \frac{40}{1} = \frac{80}{5} = 16$ $40 - 16 = 24$ 24 free seats
	36 of 1215 $\frac{36}{1215} = \frac{4}{135}$ $\frac{4}{135} \cdot 100 = \frac{400}{135} = \frac{80}{27} = 2.96\%$ $\frac{4}{135}$ of towns have it. It represents 2,96% of total towns
	15% of 96 $\frac{15}{100} \cdot 96 = \frac{1440}{100} = 14.4$ 14.4 miles.
	$\frac{5}{8}$ of 1176 ; $\frac{1}{4}$ of 1176 ; the rest $\frac{5}{8} \cdot 1176 = \frac{5880}{8} = 735$ cars $\frac{1}{4} \cdot 1176 = \frac{1176}{4} = 294$ motorbikes $1176 - 735 - 294 = 147$ other
	60.80 of 76€ ; ? of 110€ $\frac{60.80}{76} \cdot 100 = \frac{6088}{76} = 80\%$ $100 - 80 = 20\%$ off 20% of $110 = \frac{20}{100} \cdot 110 = \frac{2200}{100} = 22€$ He will pay 22€

3. Create new word problems

Instructions	Working out & solution

Session 3

Activity 3.1

SEQUENCING

This is an activity to:

- Put in order the directions of a recipe in a flow diagram and find the intruder.
- Use language to read and understand instructions.
- Use language to discuss the right order of a process.
- Watch and listen a video about a similar recipe to reinforce language.

INSTRUCTIONS

- Work in pairs
- Read the pieces of recipe and put them in order.
- Discard the intruder
- Rephrase the process in a flow diagram
- While and after watching the video, answer some questions about it.

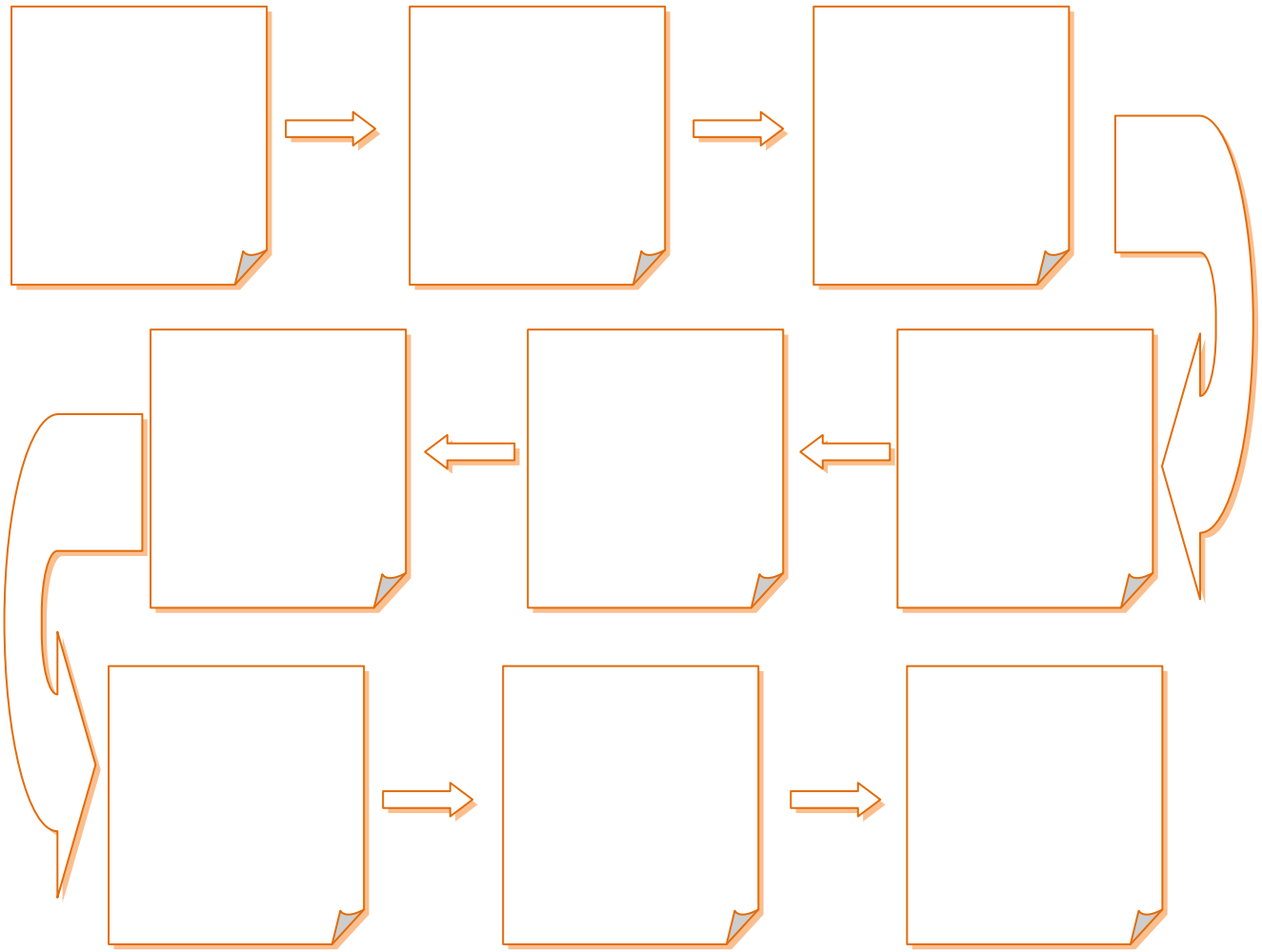
Check dictionary for new words
English only!

1.

Brownie Cheesecake

Ingredients
1 cup flour 1 cup old-fashioned rolled oats 1/2 teaspoon baking powder 1/3 cup packed light brown sugar 1/2 cup butter 1 3/4 cups blueberry pie filling (or any other flavour pie filling) 16 ounces cream cheese, softened 1 cup white granulated sugar 1 1/2 teaspoons pure vanilla extract 2 cups sour cream 3 eggs
Preparation
Press all but 3/4 cup of the crumb mixture into the bottom of a 10 inch round baking pan.
Bake at 325 degrees F for 50-60 minutes. Remove from oven, cool to room temperature and refrigerate overnight.
Add the vanilla and sour cream and beat until smooth. Beat in the eggs, mixing well after each addition.
Spread the pie filling over the top and sprinkle with the remaining crumb mixture.
Take spoonfuls of the meat mixture and form them into balls by rolling them between your palms or, if you feel like impressing the children, tossing them lightly from one palm to another to tighten the mixture.
Preheat oven to 350 degrees F (175 degrees C).
Pour into prepared crust.
In a medium bowl, combine the flour, oats, baking powder, brown sugar and butter.
In a large mixing bowl, beat the cream cheese and sugar on low speed until well blended.
Bake at 350 degrees. F (175 degrees C) for 15 minutes. Remove from oven and let cool for 10 minutes.

2. Flow diagram



3. Watch the cookery video where a cheesecake recipe is shown. Make the list of ingredients and summarize the process.

INGREDIENTS	PROCESS

Session 3

Activity 3.2

ORDERING

This is an activity to:

- Convert different expressions (fractions, decimal and mixed numbers) and different units (grams, ounces, pounds and different spoon sizes) to the same form. Finally, to order them according to their value along in a number line.
- Find and express the proportion of each ingredient in a pie chart

INSTRUCTIONS

- Work in pairs
- Read ingredients of recipe again. Notice that they are given in different units and measures.
- Convert mixed numbers to fractions
- Use the conversion table to get the ingredients in grams.
- Write the ingredients, fraction, ounces and grams in the grid below.
- Put the ingredients in increasing order on the number line
- Write the proportion of each ingredient in the whole cake as a percentage and represent it on a pie chart
- Individually (homework) do exercise 5 and 6.

Brownie Cheesecake

Ingredients
1 cup flour
1 cup old-fashioned rolled oats
1/2 teaspoon baking powder
1/3 cup packed light brown sugar
1/2 cup butter
1 3/4 cups blueberry pie filling (or any other flavour pie filling)
16 ounces cream cheese, softened
1 cup white granulated sugar
1 1/2 teaspoons pure vanilla extract
2 cups sour cream
3 eggs

1. Complete

Ingredients	Quantity	Fraction	Ounces/Pounds	Grams
Flour	1 cup	1	4.4 oz	124.74 g

- Conversion table

1 Ounce	28.35 grams
1 pound	450.6 grams
1 cup of flour	4.4 ounces
1 cup of butter	0.5 pounds
1 cup brown sugar	0.485 pounds
1 cup white gran. sugar	7.06 ounces
1 cup sour cream	8.11 ounces
1 cup rolled oats	1.41 ounces
1 cup pie filling	262 grams
1 teaspoon baking powder	4.6 grams
1 teaspoon vanilla extract	4.2 grams
1 egg	1.16 ounces

2. Number line

Decide how long the unit will be. Example: 20 g = 1 cm

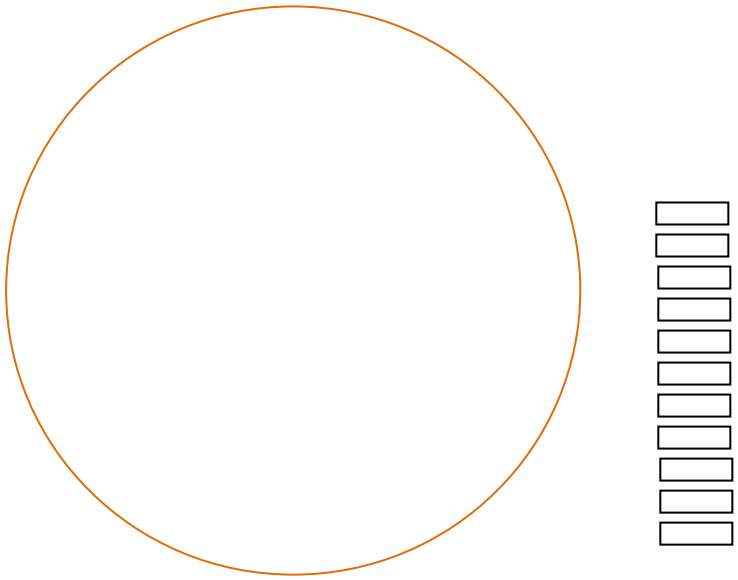
0

3. Fractions of each ingredient

Complete the grid finding the fraction represented by each ingredient.

Ingredients Increasing order	**Fraction (weight /TW)**	**%** **(Fraction ·100)**	**Degrees** **(Fraction ·360)**
Total weight (TW):	**1**	**100%**	**360º**

4. Draw a pie chart showing cake ingredients



5. Fill the gaps with the following words:

Grams, most, two, ingredient, same, sugar, addition weight, cheese, more, quantity

1. The _____ that has the lowest weight is baking powder
2. The cake contains more _____ of flour than of eggs
3. Sour cream is the _____ abundant ingredient
4. There is nearly the _____ quantity of sour cream than of blueberry pie filling
5. There are more than 100 _____ of butter
6. The cake contains more than double of white granulated _____ than of brown sugar
7. Only _____ foods weigh less than 10 grams.
8. The total weight of the cheesecake is the _____ of ingredients weight.

6. Look for another cheesecake recipe on the Internet and compare it with this one. Notice what differences and similarities there are (concerning to the ingredients) and write them down in this grid.

Differences	Similarities

English only!

Session 3

Activity 3.3

CHANGING SERVINGS

This is an activity to:

- Learn how to increase or decrease quantities in a recipe according to the servings.
- Rewrite simple recipes changing number of servings
- Match pairs, threes, fours, ... of proportional numbers

INSTRUCTIONS

- Work in pairs
- Construct a matching game using the table in next page.
- Below, there are the main ingredients of four recipes (pancakes, crêpes, rolls and scones). All of them have the same ingredients, but in different proportion.

A 9 pancakes	B 12 crêpes	C 6 rolls	D 4 scones
¼ cup sugar	½ cup sugar	½ cup sugar	½ cup sugar
½ cup flour	1 ½ cups flour	5 cups flour	3 cups flour
½ cup milk	2 cups milk	1 cup milk	1 cup milk
3 eggs	2 eggs	2 eggs	3 eggs

- Using equivalent fractions, calculate ingredients in each recipe for a different amount of units. Example:

$$\frac{9\ pancakes}{3\ eggs} = \frac{3\ pancakes}{1\ egg} = \ldots \quad ; \quad \frac{9\ pancakes}{\frac{1}{2}\ c.suggar} = \frac{3\ pancakes}{\frac{1}{6}\ c.suggar} = \ldots$$

- Write each new recipe in a cell of the table (as many as the number of cells).
- Swap the table with another pair of peers.
- Identify every cell with a recipe (A,B,C or D) and colour it as the sample. Swap tables again and check the answers. Mark your peer's work.

1.

NAMES:			MARK ☐/16
EXAMPLE **1 cup sugar** **3 cups flour** **4 cups milk** **4 eggs**			

English only!

Session 4

Activity 4.1

RUNNING DICTATION

This is an activity to:

- Memorise, read and repeat information containing fractions, proportions and percentages.
- Share and organize information.
- Make logical deductions or calculations to get some data.
- Work in groups

INSTRUCTIONS

- Work in threes
- On three places of the classroom there are three different texts (stuck on walls) with some information containing rational numbers. One student in each team runs to the text, reads it, memorises it and then dictates it to the rest of the team with as many attempts as necessary. The others in the team do the same in turns with a second and third text.
- Each one in the group keeps one of the three texts.
- Read the text carefully and be ready to explain it to the rest of group and to organise and classify the information.

1. Running dictation

Text 1

Text 2

Text 3

Session 4

Activity 4.2

ORGANISE INFORMATION

This is an activity to:

- Understand information and answer some questions about it.
- Deduce some proportions and percentages from the context.
- Rewrite form fractions to percentages or vice-versa
- Decide how to present information in order to be useful, clear and easy to find and to work with.
- Work in group

INSTRUCTIONS

- Work in threes
- After sharing the information contained in the three texts, complete the next page grid and questions. You will find answers either directly or by deducing it from the data. Work in a group and explain to each other how you have got your answers.
- Think of a diagram to present the information in a clear and useful way (there is an example in case you don't agree or you can't find a successful one).
- Read texts again and check that all the information is included.

1. Collecting and organising information

Groups

Gender	BOYS
Transportation	
	FOOT & CYCLE
Level	S1 - S4

Gender:

Percentage of boys:		**Percentage of girls:**	
Fraction of boys:		**Fraction of girls:**	
Fraction of pupils in S5-S6 being girls:		**Fraction of pupils in S5-S6 being boys:**	
Percentage of pupils in S5-S6 being girls:		**Percentage of pupils in S5-S6 being boys:**	

Transportation:

Fraction of pupils arriving by bus or car:		**Fraction of pupils arriving by foot or cycle:**	
Percentage of pupils arriving by bus or car:		**Percentage of pupils arriving by foot or cycle:**	
Fraction of boys arriving by bus or car :		**Fraction of boys arriving on foot or by bicycle :**	
Percentage of boys arriving by bus or car :		**Percentage of boys arriving on foot or by bicycle :**	
Percentage of pupils arriving by bus or car and being in S5 or S6, that are boys		**Percentage of pupils arriving by bus or car and being in S5 or S6, that are girls**	
Fraction of pupils arriving by bus or car and being in S5 or S6, that are boys		**Fraction of pupils arriving by bus or car and being in S5 or S6, that are girls**	
Fraction of pupils arriving by bus or car being in S1-S4:		**Fraction of pupils arriving by bus or car being in S5-S6:**	
Percentage of pupils arriving by bus or car being in S1-S4:		**Percentage of pupils arriving by bus or car being in S5-S6:**	

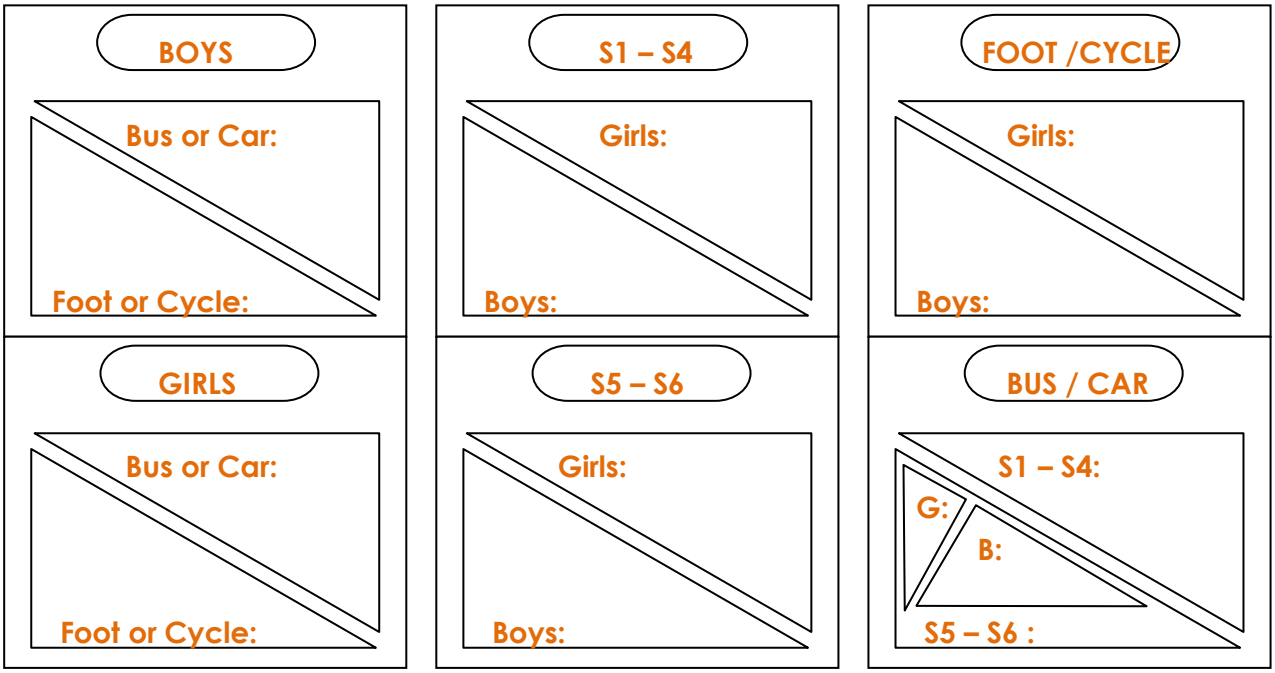
Level :

Fraction of pupils in S5 - S6:		**Fraction of pupils in S1 - S4:**	
Percentage of pupils in S5 -S6:		**Percentage of pupils in S1-S4:**	
Fraction of pupils in S1 – S4 being girls:		**Fraction of pupils in S1 – S4 being boys:**	
Percentage of pupils in S1 – S4 being girls:		**Percentage of pupils in S1 – S4 being boys:**	
Fraction of girls in S1-S4 :		**Fraction of girls in S5-S6 :**	
Percentage of girls in S1-S4 :		**Percentage of girls in S5-S6 :**	

2. DIAGRAM

SAMPLE OF DIAGRAM

(Decide if you write percentages or fractions)



Session 4

Activity 4.3

HOW MANY IN EACH GROUP?

This is an activity to:

- Reorganise information from the previous activity.
- Knowing the whole, calculate fractions of it.
- Justify answers (showing calculations) to your partners and be ready to justify to the plenary or teacher.
- Fill a grid with answers
- Work in group

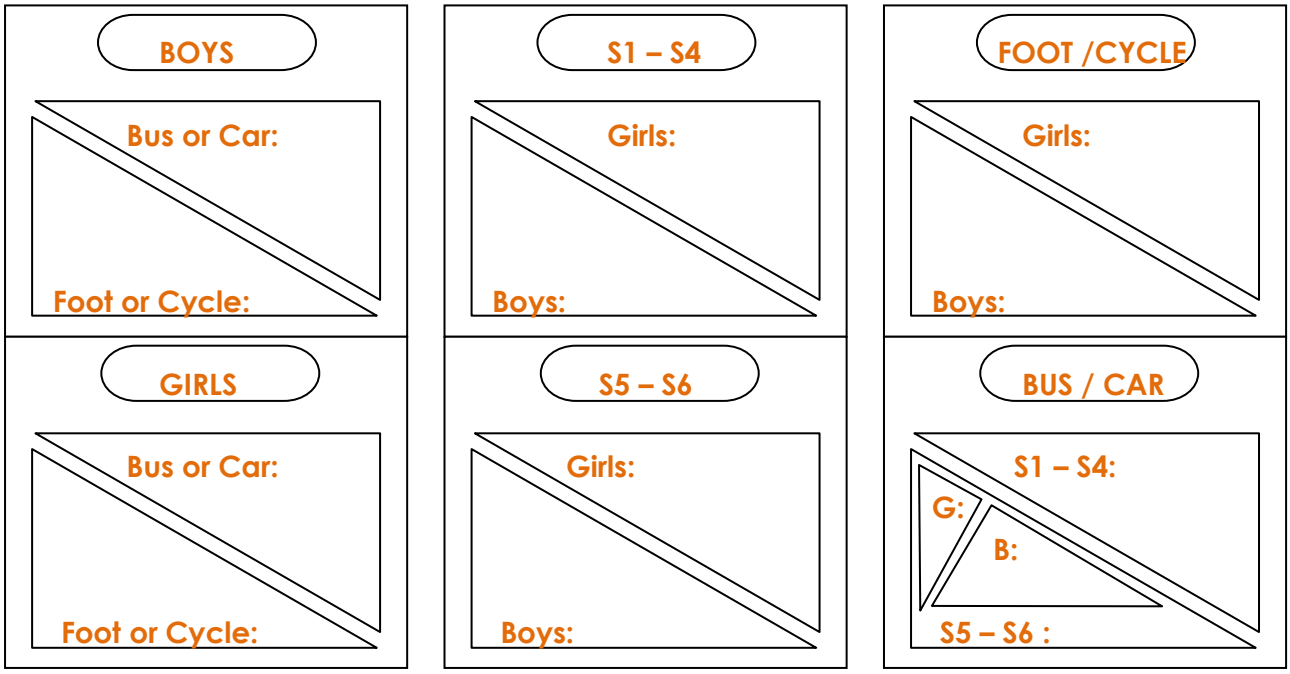
INSTRUCTIONS

- Work in threes
- Using the information included in the diagram 4.2, and knowing that there are **500** pupils in Hazlehead Academy, calculate:
  - Figures
  - Totals (helpful to check if answers are correct)
  - Fractions
  - Percentages

  to complete the following tables.
- Notice that totals are additions of rows or columns and must equal to:
  - 1 if you are adding fractions
  - 100 if you are adding percentages
  - The total of pupils, if you are adding figures
- Individually (homework)
  Analyse data and do exercises 3 and 4

1. Knowing that the number of pupils in Hazlehead Academy is 500, find how many pupils are in each group.

First, it could be useful to refill the previous diagram with absolute numbers (number of pupils in each group). So, calculate it using the other diagram and the number of pupils of the academy.



In the next grid, complete firstly the "School" column and row. Secondly, complete the "Total" column and row (some figures must to be deduced by subtracting). Finally, fill the rest.

Boys	Girls

Figures	Bus&Car		Foot&Cycle		Total		School
S1 – S4							
S5 – S6							
Total							
School							

2. From the "Figures" table, get data to complete the next two tables:

Fractions over the total number of pupils: $\dfrac{\text{Pupils fulfilling the condition}}{\text{All the pupils}}$

Fraction	Bus&Car		Foot&Cycle		Total		School
S1 – S4							
S5 – S6							
Total							
School							

Percentage over total pupils: $\dfrac{\text{Pupils fulfilling the condition}}{\text{All the pupils}} \cdot 100$

Percentage	Bus&Car		Foot&Cycle		Total		School
S1 – S4							
S5 – S6							
Total							
School							

Two surprising results you wish to highlight:

- _____

_____

- _____

3. Put a cross in the appropriate square (true or false) and justify the answer.

T	F	Statement
		1. Half of pupils in S1 – S4 are girls
Justification		
		2. Looking only at pupils arriving at school by bus or car, we can observe than there are more girls than boys.
Justification		
		3. In Hazlehead Academy most pupils are girls
Justification		
		4. The percentage of boys in S1 – S4 is bigger than the percentage of boys in S5 – S6
Justification		
		5. Girls arrive at school by bus or car in greater number than on foot or bicycle
Justification		
		6. In S5 – S6 the number of boys is double that the number of girls
Justification		
		7. The proportion of boys arriving at school on foot or by cycle $\left(\frac{\text{Boys foot or cycle}}{\text{Total Boys}}\right)$ is smaller than the proportion of girls arriving at school in the same way $\left(\frac{\text{Girls foot or cycle}}{\text{Total Girls}}\right)$
Justification		
		8. The majority of pupils in S5 – S6 arrive at school by bus or car.
Justification		

Session 4

Activity 4.4

MAKE & SELECT A DIAGRAM

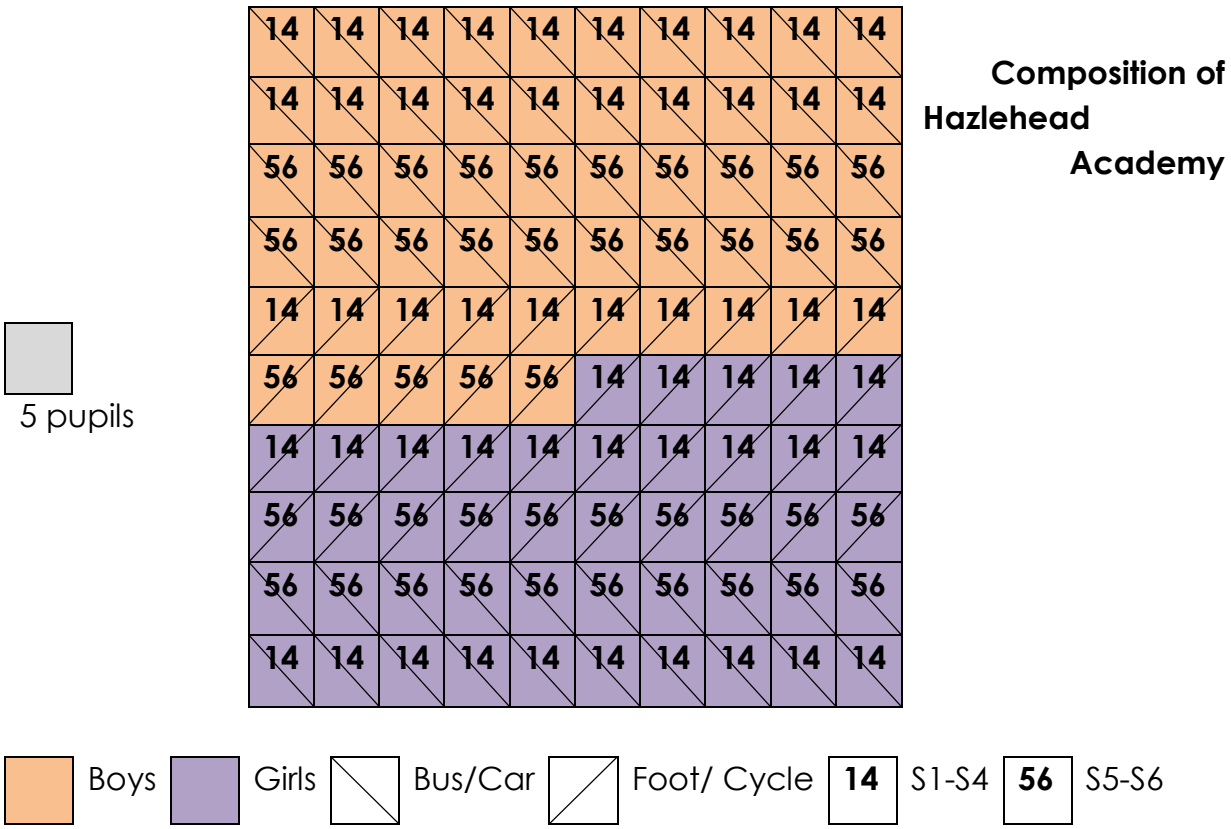
This is an activity to:

- Represent the distribution of pupils according to the criteria of gender, transportation and level.
- Use diagrams to express visually that distribution.
- Evaluate different kinds of diagrams and select the better one.
- Justify your selection according to the criteria used.
- Work individually

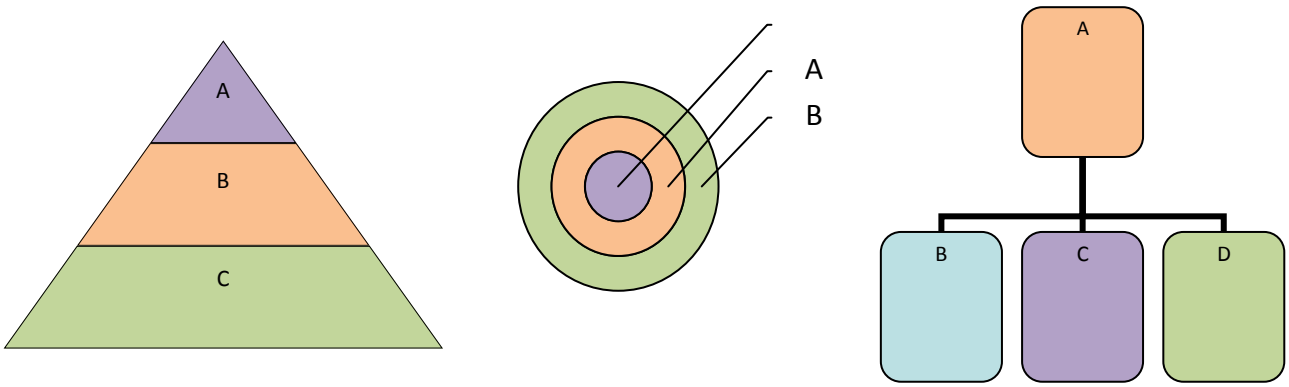
INSTRUCTIONS

- Work individually O
- Using the information included in the 4.3 grids (figures, fraction or percentage), design a diagram to represent the distribution of pupils in that school.
- To help you, look at the example.
- Once all the pupils have finished their diagrams, they will be collected (or printed) and the group will arrange an improvised exhibition.
- Look at the diagrams and evaluate them (you can use the model)
- Vote for what you think is the best one (the election will be a show of hands and it must be justified) Ø
- Individually (homework)
  Analyse data and do exercises 3 and 4

4. Look at this example of diagram:



5. Design your own diagram



6. Evaluate your peer's diagrams. Use this model if you think it can be helpful.

1. Title

Yes	No

2. Correct and complete information

1	2	3	4	5	6	7	8	9	10

3. Can you see, for example, how many or what percentage of pupils in S1-S4 are boys?

Yes	No

4. Easy to "read"

1	2	3	4	5	6	7	8	9	10

5. Design

1	2	3	4	5	6	7	8	9	10

6. Presentation

1	2	3	4	5	6	7	8	9	10

Group marks:

Names	**1**	**2**	**3**	**4**	**5**	**6**

Best Diagram: _____

Justification: _____

Session 4

Activity 4.5

COMPARING SCHOOLS

This is an activity to:

- Analyse statistics of two schools and compare them in terms of proportion of boys/girls, means of transport and educational levels.
- Notice and evaluate differences and similarities
- Hypothesise about the reasons for those differences or similarities
- Perform a debate in front of the rest of the class

INSTRUCTIONS

- Work in threes
- Look at the statistics from the Catalan IES and British academy (from activity 4.3)
- Calculate percentages of each group with respect to the total of pupils
- Compare percentages and classify them into three groups: if they are greater, equivalent or smaller in the British academy than in the Catalan IES.
- Think about the different circumstances of both schools (ask for information - or search on the internet- about climate; educational system; location, size, school year and timetable of British academies) and try to give some reason for statistics.
- Prepare to play a role (journalist or headteacher) in an interview and to discuss the statistics and possible reasons for them in your opinion.
- Record the performance
- Transcribe your piece of recording and try to improve it (homework)

1. Observe the distribution of pupils in a Catalan School (IES) shown in the grid below and answer the following questions:

Boys	Girls

Figures	**Bus&Car**		**Foot&Cycle**		**MotorBike**		**Total**		
ESO1 -ESO4	90	115	75	60	20	15	185	190	375
BAT1 -BAT2	10	20	15	10	10	10	35	40	75
Total	100	135	90	70	30	25	220	230	450
	235		160		55		450		

1. Is there the same number of pupils than in the British academy? | Yes | No |
2. Is it possible to compare directly both distributions? Why? | Yes | No |

_____

_____

3. Highlight one aspect of this school in comparison with the other :

_____

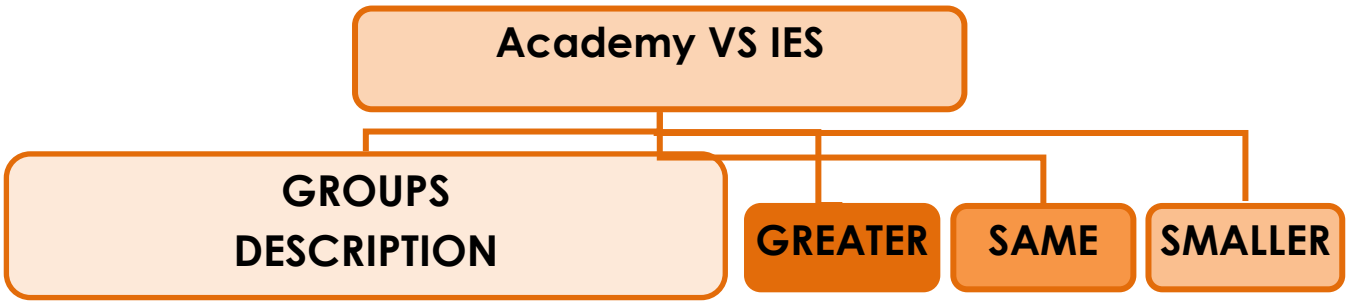
2. To compare both distributions, we need percentages (or fractions). Calculate them and put it in the next grid.

Percentage	**Bus&Car**		**Foot&Cycle**		**MotorBike**		**Total**		
ESO1 -ESO4									
BAT1 - BAT2									
Total									

3. Compare the percentages and classify then in three groups: **greater**, **same** or **smaller.**

1.	**Boys**						
2.	**S1-S4**						
3.	**Bus & Car**						
4.	**Boys ∪ Bus & Car**						
5.	**Girls ∪ Bus & Car**						
6.	**Boys ∪ Foot & Cycle**						
7.	**Girls ∪ Foot & Cycle**						
8.	**Boys ∪ S1 – S4**						
9.	**Girls ∪ S1 – S4**						
10.	**Boys ∪ S5 – S6**						
11.	**Girls ∪ S5 – S6**						
12.	**Bus&Car ∪ S1–S4 ∪ Boys**						
13.	**Bus&Car ∪ S1–S4 ∪ Girls**						
14.	**Bus&Car ∪ S5–S6 ∪ Boys**						
15.	**Bus&Car ∪ S5–S6 ∪ Girls**						
16.	**Foot&Cycle ∪ S1–S4 ∪ Boys**						
17.	**Foot&Cycle ∪ S1–S4 ∪ Girls**						
18.	**Foot&Cycle ∪ S5–S6 ∪ Boys**						
19.	**Foot&Cycle ∪ S5–S6 ∪ Girls**						

"A ∪ B" means the union of both conditions, A and B


Academy VS IES
GROUPS DESCRIPTION
GREATER
SAME
SMALLER

4. Think and speculate about the reason for the greatest differences between both schools.

Difference	Possible reasons

5. Imagine we are in a radio studio recording a programme called "Schools around the world" and you are:

- **A: A journalist**
- **B: The headteacher of the British Academy**
- **C: The headteacher of the Catalan IES**

Prepare a 5' performance pretending you are the three characters A, B and C. You need to think about a:

- Short introduction about the theme and the character
- Some questions and answers comparing both schools, evaluating differences and similarities and hypothesizing and debating the reasons (focus only in two or three aspects!)
- Conclusion and ending the interview.

NOTES
Character:
- - - - - - - - -

6. The radio programme will be recorded. Listen to your piece of speech and transcribe it. Highlight the mistakes and correct them.

TRANSCRIPTION
Character:

Session 5

Activity 5.1

PYTHAGORAS AND NUMBERS

This is an activity to:

- Search for information on the Internet
- Read and understand information from a text
- Ask and answer questions to share different pieces of information.
- Introduce irrational numbers
- Link new knowledge with previous: Update and complete.

INSTRUCTIONS

- Work individually at home
- On the internet, look for information about:
  - Irrational numbers
  - Pythagoras' works on Geometry
  - Pythagoras and the discover of irrational numbers (incommensurable numbers)
  - Pythagoras and music
- Do exercises 1 and 2 based on your research (homework)
- Check answers with your neighbour
- Read the corresponding text (there are eight different texts and your teacher will give you one of them)
- Teacher distributes questionnaires with 5 questions to every pupil
- Stand up and move around the classroom to find someone who can answer your questions.
- Share and correct answers in plenary and try to contribute with some additional information you got from your personal research or from texts.

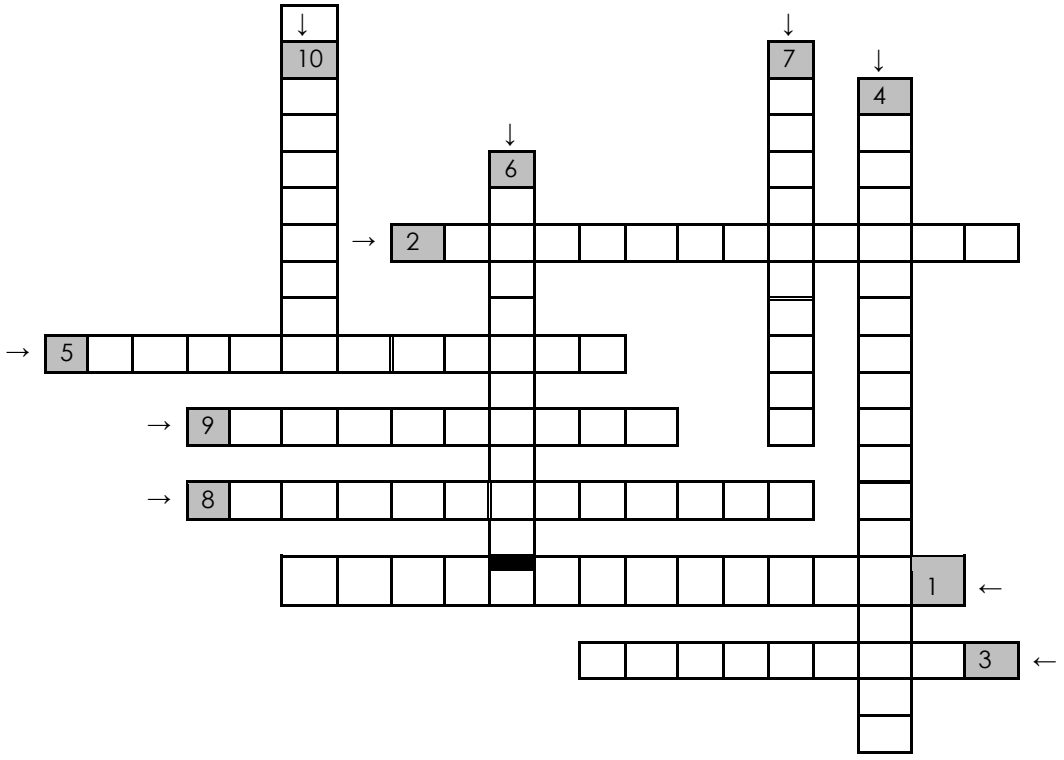
7. Match the beginnings and ends of the sentences. Then, classify sentences into four groups, according to the topic.

1	The **pentacle** symbolized	Hides the **Divine Proportion**.	A
2	When Pythagoras realized that the diagonal of a square whose side was one unit	is 'Number', and harmony can be understood in terms of simple **ratios** of the numbers 1, 2, 3, and 4.	B
3	When **irrational numbers** are expressed as decimals,	Harmony of body and mind and thus, health.	C
4	Pythagoras found that in musical instruments, strings whose lengths are related by rational ratios	Could not be expressed as a **whole number** or **fraction**, he tried to keep the secret.	D
5	**Pi** and **Phi** are famous irrational numbers	Cannot be represented as a **ratio** of integers.	E
6	**Octave** ratio 1:2 means that a string will produce a frequency an octave above the pitch of its full length	By using musical intervals to describe the distances between the moon and the known planets	F
7	The discovery of **irrational numbers**	Because it was probably the first **irrational number** they were aware of.	G
8	**Irrational** mean that it cannot be expressed as a **ratio**,	Allows us to find the length of the third **side**.	H
9	Pythagoreans believed that all of reality, in its underlying essence,	And Pythagoreans, traumatised by the discovery, threw him overboard.	I
10	$\sqrt{2}$ is called the Pythagoreans' constant	Caused a significant re-evaluation of Greek philosophy of mathematics	J
11	The **pentagon star** formed by the five diagonals of a regular pentagon	They are **non-terminating** and **non-recurring.**	K
12	The funny thing is that the concept of irrationality	Represented by the symbols **π** and **φ**.	L
13	**Irrational number** is a number that	Have **harmonious** pitches	M
14	Legend told that **Hippassus** showed that the diagonal of a unit square was an **irrational number**	was implicitly accepted by Indian mathematicians since the 7th century BC, one century before Pythagoras	N
15	Pythagoras devised a literal "**music of the spheres**"	When it is reduced by one half	O
16	If we know the lengths of two **sides** of a **right triangle**, **Pythagoras' Theorem**	Not that it is crazy!	P

Irrational numbers	Pythagoreans & Geometry	Pythagoras & irrational numbers	Pythagoras, music and philosophy
			1-C,

8. Solve the crossword. Some answers are two words and there is a double line to indicate this.

1. Pythagoras of Samos is often described as the first pure _____.
2. He founded a secret society in Croton. His followers were called _____.
3. Members of this society used the _____ symbol in order to identify one another and to represent health.
4. The discovery of _____ _____ is attributed to the Pythagoreans, although one of their principles was that whole numbers and fractions could describe anything.
5. In the pentacle, each intersection divides the side into two parts with a proportion called the _____ _____ or divine proportion.
6. In any right triangle, the square of the _____ equals the sum of the squares of the other two sides.
7. According to legend, Hippassus proved the irrationality of the _____ _____ of two.
8. The sound produced by the seven planets going around the Earth was called the _____ of _____.
9. Pythagoras studied the _____ and discords resulting from combinations of sounds and discovered that the proportion was the key.
10. By experimentation, he discovered the harmony of the _____ when the ratio between two strings was 3:2.



9. Read one of the texts (1 to 8).

10. Take a questionnaire and ask your peers for answers. Write your questions and answers here. Then, in plenary, check answers and make some notes if your teacher or other pupils provide additional information.

n	QUESTION	ANSWER
1.		
2.		
3.		
4.		
5.		

11. Summarize questions from others questionnaires

n	QUESTION	ANSWER
6.		
7.		
8.		
9.		
10.		
11.		
12.		
13.		
14.		
15.		
16.		
17.		
18.		
19.		
20.		

Session 6

Activity 6.1 - 6.2

OPERATIONS

This is an activity to:

- Learn how to operate fractions and mixed numbers
- Calculate multiple operations
- Learn the language of operations
- Use language to explain a procedure at the same time that you develop it

INSTRUCTIONS 1

- Work in pairs
- After listening to four short clips about how to calculate operations with fractions (examples of how to work out the addition, subtraction, multiplication and division of fractions) you are going to listen to other short videos. This time, the clips will be without sound and they will show how to solve an operation on a board.
- Write the script for those videos using the language learnt in this activity.
- Practice it with your partner and then you will perform it in front of the class

INSTRUCTIONS 2

- Work individually
- Do the following exercises involving single or multiple operations.
- Compare results with your peers
- Correct them on the board (each student solve at least one multiple operation while he/she explains it step by step)

Activity 6.1

1. Watch the clip without sound and write the script. Try to explain every step of the procedure at the same time as frames are screened.

N	**Mathematical language**	**SCRIPT**
1.		
2.		
3.		
4.		
5.		
6.		

Activity 6.2

1. Find the missing numbers

$$\frac{\square}{7}-\frac{-2}{7}=\frac{14}{\square}=\frac{\square}{1}$$

$$\frac{1}{3}+\frac{\square}{8}=\frac{\square}{24}+\frac{27}{24}=\frac{\square}{24}$$

$$\frac{6}{5}\cdot\frac{-30}{\square}=\frac{-180}{55}=\frac{\square}{11}$$

$$\frac{-4}{\square}:\frac{1}{8}=\frac{\square}{10}=\frac{\square}{\square}$$

$$\left(\frac{-3}{2}\right)^{3}=\frac{\square^{3}}{\square^{3}}=\frac{\square}{8}$$

$$\frac{3}{8}+\frac{-1}{\square}:\frac{4}{\square}=\frac{\square}{8}+\frac{-9}{8}=\frac{-6}{\square}=\frac{\square}{\square}$$

$$\frac{-7}{\square}\cdot\frac{1}{6}\cdot\frac{-3}{2}-\frac{\square}{2}:\frac{\square}{7}=\frac{\square}{24}-\frac{21}{12}=\frac{\square}{\square}-\frac{\square}{24}=\frac{\square}{24}=\frac{\square}{8}$$

$$\square-\left(\frac{-13}{18}+\frac{5}{6}:\frac{\square}{10}\right)^{\square}=\frac{3}{1}-\left(\frac{\square}{\square}+\frac{\square}{18}\right)^{2}=\frac{\square}{1}-\left(\frac{\square}{18}\right)^{2}=\frac{3}{1}-\frac{\square}{324}=\frac{\square}{324}-\frac{\square}{324}=\frac{\square}{324}$$

2. Calculate, writing the answer in its simplest terms

$\frac{-4}{3}-\left(\frac{-1}{5}+2\right)\cdot\frac{-5}{6}=$ $--\left(-+-\right)\cdot-=$ $---\cdot-=---$ $---=-=$	$-3^{3}:\left(\frac{-1}{6}\right)^{2}\cdot\frac{5}{12}=$ $-:-\cdot-=$ $-\cdot-=-=$

$\sqrt{\frac{9}{25}}-\left(\frac{4}{6}\right)^2\cdot 3^3=$ $---\cdot-=---=$ $=---=-=$	$\frac{2}{5}:\frac{-7}{12}+\frac{-1}{3}\cdot\frac{15}{4}=$
$\frac{3}{8}-\frac{13}{6}:\frac{-7}{12}-1=$	$\frac{-9}{4}\cdot\left(\frac{1}{2}-3\right)+\left(\frac{-4}{3}\right)^2=$
$\frac{-2}{9}-\frac{3}{16}:\frac{-2}{10}+1=$	$\frac{3}{7}:\frac{-5}{12}+\frac{5}{6}\cdot\frac{-5}{4}=$
$\frac{-3:\frac{5}{12}}{\frac{1}{3}+2\cdot\frac{-14}{3}}=$	$\frac{-6}{4}\cdot\left(\frac{1}{3}+5\right)+\left(\frac{-5}{4}\right)^2=$
$\frac{-4:\frac{6}{10}}{\frac{1}{2}+3\cdot\frac{-5}{2}}=$	$\frac{\left(\frac{6}{7}\right)^2+1}{\frac{-1}{5}:\frac{3}{10}+\frac{-4}{5}}=$

3. Choose the correct answer. Justify it.

QUESTION		JUSTIFICATION
1. Calculate $\frac{2}{x}+\frac{3}{x}$		
○ $\frac{6}{x^2}$ ○ $\frac{6}{x}$	○ $\frac{5}{x}$ ○ $\frac{5}{x^2}$	
2. Calculate $\frac{y}{4}\cdot\frac{7}{3}$		
○ $\frac{7y}{12}$ ○ $\frac{3y}{28}$	○ $\frac{7+y}{12}$ ○ $\frac{4y}{21}$	
3. Calculate $\frac{1}{3}-\frac{z}{5}$		
○ $\frac{1-z}{-2}$ ○ $\frac{1-z}{15}$	○ $\frac{3-5z}{15}$ ○ $\frac{5-3z}{15}$	
4. Calculate $\frac{3}{x}:\frac{6}{y}$		
○ $\frac{y}{2x}$ ○ $\frac{6x}{3y}$	○ $\frac{18}{xy}$ ○ $\frac{3x}{6y}$	
5. Calculate $\frac{2}{a}+\frac{7}{b}$		
○ $\frac{2b+7a}{ab}$ ○ $\frac{9}{ab}$	○ $\frac{9}{a+b}$ ○ $\frac{2a+7b}{ab}$	
6. Calculate $\left(\frac{-2}{a}\right)^3$		
○ $\frac{-8}{a}$ ○ $\frac{-6}{a^3}$	○ $\frac{-8}{a^3}$ ○ $\frac{-6}{3a}$	

Session 9

Activities 7.1- 7.2- 7.3

WORD PROBLEMS

This is an activity to:

- Identify, solve and classify problems involving operations of fractions
- Listen to short word problems, understand and solve them mentally.
- Set out new problems based on examples and send to the class using "Twitter".

INSTRUCTIONS 1

- Work in pairs
- There are 16 cards containing solved problems, only instructions, only solutions or nothing at all (blank cards). Problems can be classified as: "Problems involving only fractions of the whole" or "Problems involving fractions of the remaining part". To start playing, one student takes a card,
  - If it is a solved problem, s/he reads it and they both classify it
  - If the card contains the instructions of a problem, both students try to solve it and then, they classify it.
  - If the card contains only the solution, they set out a problem which could match the answer.
  - If it is a blank card, create a new problem that could be classified in one of two groups.
- Check answers in plenary

INSTRUCTIONS 2

- Work individually
- Your teacher will read (twice) some short problems involving only one or maximum two operations.
- Find the solution mentally and write the answer in the grid below
- In the second exercise of mental computation, you must fill the diagram with the answers. Calculate them mentally. Don't use a calculator.
- Correct in plenary

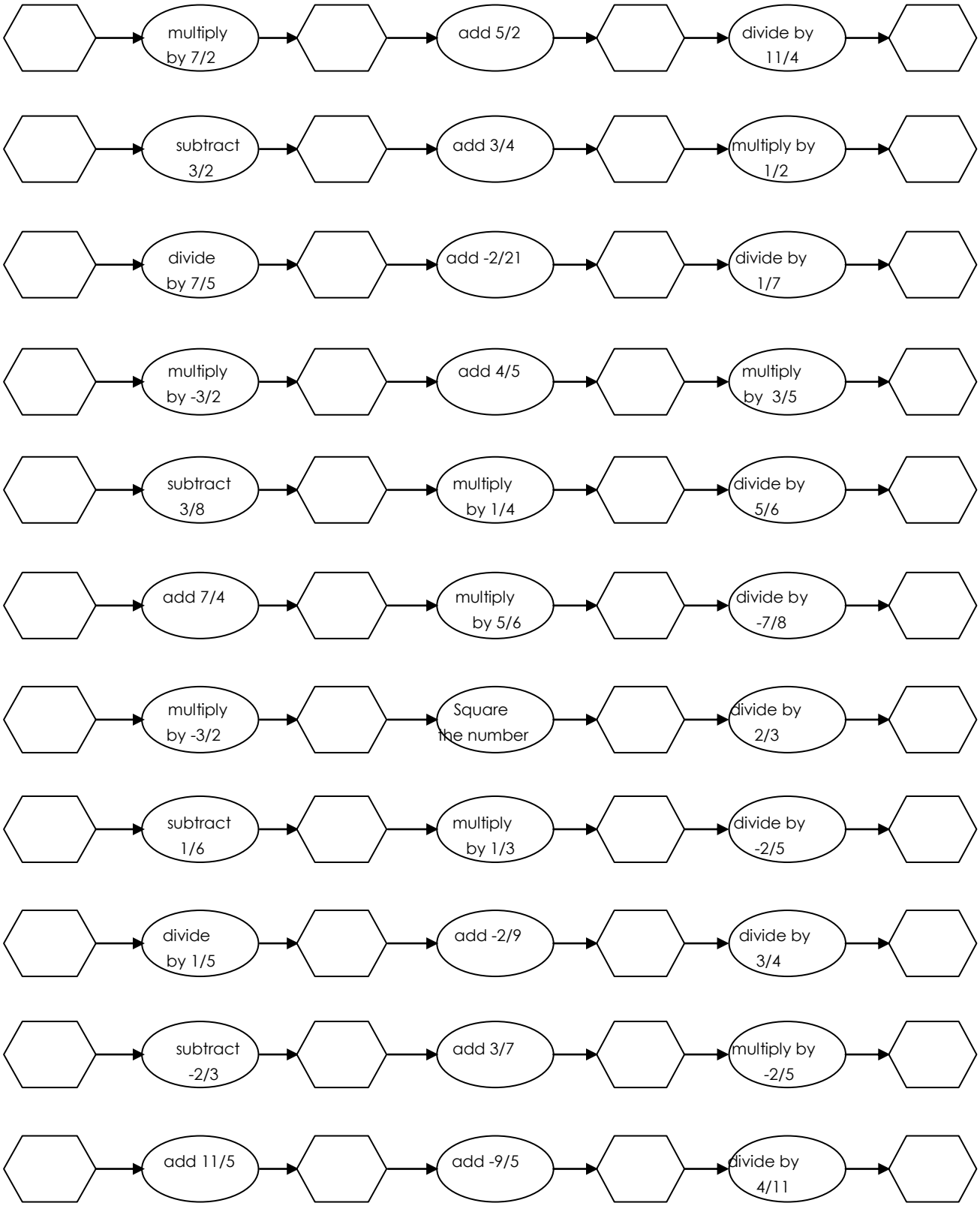
1. Solve and classify the problems from the cards

2. Fill the grid below

Notes	Solution
1.	
2.	
3.	
4.	
5.	
6.	
7.	
8.	
9.	
10.	
11.	
12.	

13.	
14.	
15.	
16.	
17.	
18.	
19.	
20.	
21.	
22.	
23.	
24.	

3. Complete the following chains of operations



Session 10

Activity 8.1 – 8.2

GEOMETRIC PROBLEMS

This is an activity to:

- Review geometric proportionality: the Thales theorem and similar shapes
- Review geometry: area and surface formulas and trigonometry relationships.
- Solve problems applying given methods.
- Use language to describe a geometric drawing.
- Use language to understand word problems and to create new ones.

INSTRUCTIONS 1

- Work in pairs
- Your teacher will give you a set of cards, 10 of them containing drawings and 10 containing the instructions of a problem related to the drawing. Place them face down in two piles (instructions /drawings). The first student takes a card with a drawing and describes it to the other, avoiding showing it and expecting the partner to draw a replica. When the replica is finished, they both have to match the drawing with the appropriate instructions.
- Compare the replica with the original. Write on the drawing the side lengths, the measure of the angles and other values deduced from the instructions.
- Evaluate what relationships can be applied to the problem to lead to the solution.
- Solve the problem. Start again with a new card but swapping roles.
- Create a new problem, starting by drawing a shape with a shaded area and finishing by writing the instructions.
- Exchange it with another pair and do as homework
- Check answers and methods in plenary

INSTRUCTIONS 2

- Work individually (as homework)
- Solve the multiple choice questions. Justify the answer working it out
- Check answers next session

2. Draw what your partner is describing

Drawing 1	Drawing 2
Drawing III	**Drawing IV**
Drawing V	**Drawing VI**

3. Create a new problem

Drawing	Instructions
Answer	

4. Tricky problems. Solve and answer the following questions.

1. One melon weights 7/5 Kg more than the 5/7 of the same melon. What is the total weight?

- 14/35 Kg
- 7/10 Kg
- 35/14 Kg
- 49/10 Kg
- 5 Kg

2. A full milk churn weights 25 Kg. When the same churn is half full, the weight is 14 Kg. What percentage of the total weight (full of milk) represents the milk churn?

- 4%
- 8%
- 10%
- 12%
- More than 12%

3. Roger's mother prepared a litre of orange juice. Roger was thirsty and drank a big glass (1/4 litre) of the juice. So that his mother did not notice the missing juice, he refilled the pitcher with water. Later, Roger was thirsty again and drank another 1/4 litre glass of the mixture. He believed that refilling with water again, would make the juice taste too watery, therefore, he squeezed a quarter of litre of orange juice and added it to the pitcher. After all this, what part of the liquid in the pitcher is water?

- 7/16
- 3/8
- 5/16
- 2/8
- 3/16

4. We cut a ribbon of 84 cm in four pieces so that the first piece is the double of the second one, the second piece is three times the third one and the fourth piece is four times the third one. What length is the fourth piece?

- 36 cm
- 32 + 4/5 cm
- 24 cm
- 8 + 1/5 cm
- 6 cm

5. *S* is 25 % of 60,
60 is 80 % of *U*,
80 is *M* % of 25
What is the solution of *S* + *U* + *M*?

- 410
- 83
- 110
- 383
- 735

Session 11

Activity 9.1 – 9.2

MULTISTEP PROBLEMS

This is an activity to:

- Learn two methods to solve multistep problems involving fractions
- Solve the problems applying given methods or a new one.
- Use language to compare and contrast different strategies to solve a problem.
- Use language to understand word problems and to create new ones.

INSTRUCTIONS 1

- Work individually ○
- There are 5 problems to solve. Each one has to be solved by two different methods (arithmetic, equation, using a diagram or your own method).
- Compare solutions
- Decide which one is shorter and easier for you in each problem.
- Create 2 new multistep problems presenting real situations (data and solutions have to be feasible).
- Exchange them with another pair and do as homework
- Check answers and methods in plenary Ø

INSTRUCTIONS 2

- Work in pairs ⌀
- Solve the problem using equations and using a diagram
- Compare results
- Watch a video where the same problem is solved.
- Compare all the methods used to solve the problem and decide with your partner the most understandable, the shortest and the easiest in your opinion. Justify the election.

5. Problems to solve

a) The *Flowers Are Live Company* sends a team of three gardeners to do a job. They divide up the work proportionally to the time they have been in the company. David has only nine months of experience and has to take care of one quarter of the garden. Alison is assigned one third of the garden and James, the more experienced, must care for the remaining 240 square metres. What is the area of the garden?

Method 1	**Method 2**

b) Bruce had to roll 1000 posters into a tub. Before starting, he noticed it was 9:30 a.m. At 2:45 p.m. he counted how many posters he had folded and he calculated that he had only 2/5 of the posters left to do. He continued at the same rate. How much longer will Bruce have to work to complete the entire job?

Method 1	**Method 2**

c) Ben sold 7/12 of his poultry. Of his unsold poultry, 3/5 were chickens and the rest were 14 ducks. What fraction of the all the poultry were the unsold ducks? How many animals had he in total?

Method 1	**Method 2**

d) A third grade teacher had a box of pencils. If 1/10 of the pencils are green, ½ of them are white, ¼ of them are blue and the remaining 45 pencils are red, what is the number of blue pencils?

Method 1	**Method 2**

e) Peter owes her friend x dollars. Last month she paid ¼ of the amount owed. This month she paid her friend 1/5 of the remaining amount plus 15.00$ In terms of x, how much money does she still owe?

Method 1	**Method 2**

f) A train starts with a number of passengers. At the first station, it drops one-third of the passengers and takes 280 more. At the second station, it drops one-half of the new total and takes 12 more. On arriving at the third station, it is found to have 248 passengers. Find the number of passengers in the beginning.

Method 1	**Method 2**

g) Brian takes an 81 inch piece of rope and cuts it three times. Each time he cuts the rope, he cuts off and discards the same fraction of the remaining length. When he is finished, the piece of rope is 3 inches long. Which fraction represents the amount of rope removed in each of the three cuts? The answer is 2/3, but why

Method 1	**Method 2**

h) The gravestone of the Greek mathematician Diophantus (250 B.C.) has the following inscription:
"During a sixth part of his life he was a child. A twelfth part later he became a man and a seventh part later he married. After five years he received a son, who reached only half the age of his father. Diophantus mourned four years for the death of his son before he died."

Method 1	**Method 2**

i) A rat population is composed by 25% of white rats and 75% of black rats. Half of the white rats and the fifth part of the black rats have long tail. If we know that there are 99 rats with long tail, how many rats comprise the population?

Method 1	**Method 2**

j) Lara is training for the Boston Marathon. Yesterday she ran 7/8 of a 20 mile course in 2 hours and 48 minutes. Today Lara ran at the same speed but was able to finish the entire course. How long did it take Lara to complete 20 miles?

Method 1	**Method 2**

6. Now, you are going to watch a video where a resolution of the last problem (problem j)) is shown. Make some note and compare with your methods.

Notes on the method used on the video	
Decide which method is better in your opinion and justify your answer	
Pros	**Cons**

Session 12

Activity 10.1 – 10.2

WORKING WITH FORMULAS

This is an activity to:

- Identify formulas and their elements
- Use given formulas to find the subject, what implies to calculate numerical expressions with rational numbers, or to change the subject.
- Create functions (formulas) that model a real situation.
- Translate from sentences to algebraic language and vice-versa

INSTRUCTIONS 1

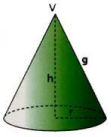
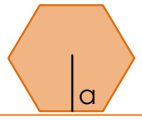
- Work in pairs
- Match geometric, arithmetical, statistical or other fields' formulas (already known) with a drawing or a theorem. Explain the meaning of each variable and what the formula is used for.
- Translate from a sentence to an algebraic expression and replace variables with the given numbers
- Given a formula and the values of all the variables except one, find the missing one. If the variable you need to calculate is the subject, just replace the values and calculate, if not, start by changing the subject. Compare solutions with your partner.
- Create a new formula (individually) and read it to your partner. Your partner has to write them and calculate the subject substituting the variables for any fractions. Swap roles.
- Check answers in plenary

INSTRUCTIONS 2

- Work individually
- Match the statements with the tables below. Complete it by writing the formula that models the situation.
- Create two new situations and fill the corresponding tables. Deduce the formula that models the situation you have invented.

7. Match one element of each column. Explain in what situations we use the formula and the meaning of the variables.

1. $V = \frac{\pi \cdot r^2 \cdot h}{3}$	
2. $E = m \cdot c^2$	
3. $I = \frac{C \cdot r \cdot t}{100}$	
4. $\bar{x} = \frac{x_1 + x_2 + x_3 + x_4}{4}$	
5. $v = \frac{s}{t}$	
6. $S = \frac{p \cdot a}{2}$	
7. $A = \frac{2 \cdot \pi \cdot R \cdot \alpha}{360^\circ}$	
8. $d = \frac{m}{V}$	

	A. Theory of relativity
	B. Arithmetic mean
	C. 
	D. 
	E. 
	F. Simple Interest
	G. 
	H. Uniform motion

Solution	Meaning of variables / Use
1-E	V= volume, r= radius, h=height It is used to calculate the volume of a cone when we know the height and the radius.

8. Translate from the text to a formula and replace the variable as in the example

TEXT	FORMULA
1. Quadruple a number plus six is a second number. If the first number is five-halves then what is the second number? **Ans**: 16	Formulas: $4n + 6 = m$ Substitution: $4(5/2) + 6 = m$ Simplification: $20/2 + 6 = m$ $16 = m$
2. Quadruple a number plus six is a second number. If the first number is three-fourths then what is the second number? **Ans**: 9	
3. Two-thirds of a number less one half is a second number. If the first number is one-forth, then what is the second number? **Ans**: -1/3	

TEXT	FORMULA
4. Two less than three-halves of a number is a second number. If the first number is ten, then what is the second number? **Ans**: 13	
5. Three-fours of a number less two thirds of that same number is a second number. If the first number is twelve then what is the second number? **Ans**: 1	
6. Five-sixths less than two-third of a number is a second number. If the first number is two fifths, then what2 is the second number? **Ans**: -17/30	
7. Two-fifths the square of a number is a second number. If the first number is three-fourths, then what is the second number? **Ans**: 9/40	
8. Bonus Question: Marco's age divided by two-thirds is equal to Billy's age. If Marco is five and a half, how old is Billy? **Ans**: 8 1/4	

9. Complete

Formula	Find the subject in both situations	Change the subject. Rearrange the equation
$y = 3.5 \cdot x + 20$ "Y equals to three point five times x plus twenty"	$x = \frac{1}{7}$	X is the new subject Read the new formula
	$x = 4\frac{1}{2}$	Now, find X when $y = 90$
$c = \sqrt{a^2 + b^2}$ "c equals the square root of the sum of squares of a and b"	$a = \frac{5}{2}$ $b = 6$	"a" is the new subject Read the new formula
	$a = \sqrt{5}$ $b = \sqrt{20}$	Now, find "a" when $b = \frac{8}{3}$ $c = \frac{17}{3}$

10. Constructing formulas

Read the situations. Match the statements with the tables below.

1. Our car is running at 50 miles per hour. We'd like to know the formula expressing the relationship between the hours (t) we are driving and the distance covered (d).
2. All the articles of a shop are reduced. They are reduced 20 % from the initial price. We would like to know the formula that relates the initial price (I) to the reduced price (R).
3. The distance between Aberdeen and Edinburgh is 120 miles. We want to know the formula relating the speed (s) with the time we spend (t).
4. A taxi charges 50p per mile plus a fixed charge of 2 pounds. Find the formula relating the cost (c) with the miles (d) covered

Create 2 new situations:

5.
6.

Tables

A- Subject:	Variable:
10	10 · 0.5 + 2 = 7
15	15 · 0.5 + 2 = 9.5
30	30 · 0.5 + 2 = 17
t, i, s or d	

B- Subject:	Variable:
10	10 · 50 = 500
15	15 · 50 = 750
5	5 · 50 = 250
t, i, s or d	

C - Subject:	Variable:
10	120 / 10 = 12
50	120 / 50 = 2.4
40	120 / 40 = 2.4
t, i, s or d	

D - Subject:	Variable:
10	10 – 10 · 20/100= 8
15	15 – 15 · 20/100= 12
50	50 – 50 · 20/100= 40
t, i, s or d	

E - Subject:	Variable:

F - Subject:	Variable:

Explain to the class how you have worked out the formula and prove that it works using examples.

Session 13

Activity 11.1

EGYPTIAN FRACTIONS

This is an activity to:

- Read and understand information from a text
- Ask and answer questions to share different pieces of information.
- Introduce notions about history of mathematics
- Write a relationship between numbers in words and with a formula.
- Play solitaire to reinforce concepts of lesson 1 and lesson 2.

INSTRUCTIONS

- Work in threes
- Your teacher will distribute three different texts about fractions in the ancient Egypt and a questionnaire.
- Choose one text and read it.
- Answer the questionnaire
- (Homework) Select an aspect of the three texts (the most curious or interesting from your point of view) and design a poster to illustrate it. You can use the computer. To print it, it is better to work on 3 or 4 sheets (A4) and then put them together.

- Work in pairs
- Read the definitions of the Leibniz and Pascal triangles.
- Work out the patterns and properties of these number triangles
- Correct them in plenary

1. Read the three texts and answer the next questionnaire

QUESTIONNAIRE
1. Define a unit fraction. Give an example
2. Is it true that every proper fraction has an Egyptian expansion? Why do we know this?
3. How do we know that Egyptians were familiar with fractions?
4. Apply the greedy algorithm to the fraction 6/7
5. Write the hieroglyphs for all the fractions involved in the Egyptian representation of question 4.

6. Write three Egyptian expansions of the fraction 5/7 and discuss which one would scribes have preferred. Write the conclusions.

2. Write the main ideas about Egyptian fractions you want to illustrate in your poster.

n	MAIN IDEAS
1.	
2.	
3.	

3. Read the definition of the Leibniz and Pascal triangles.

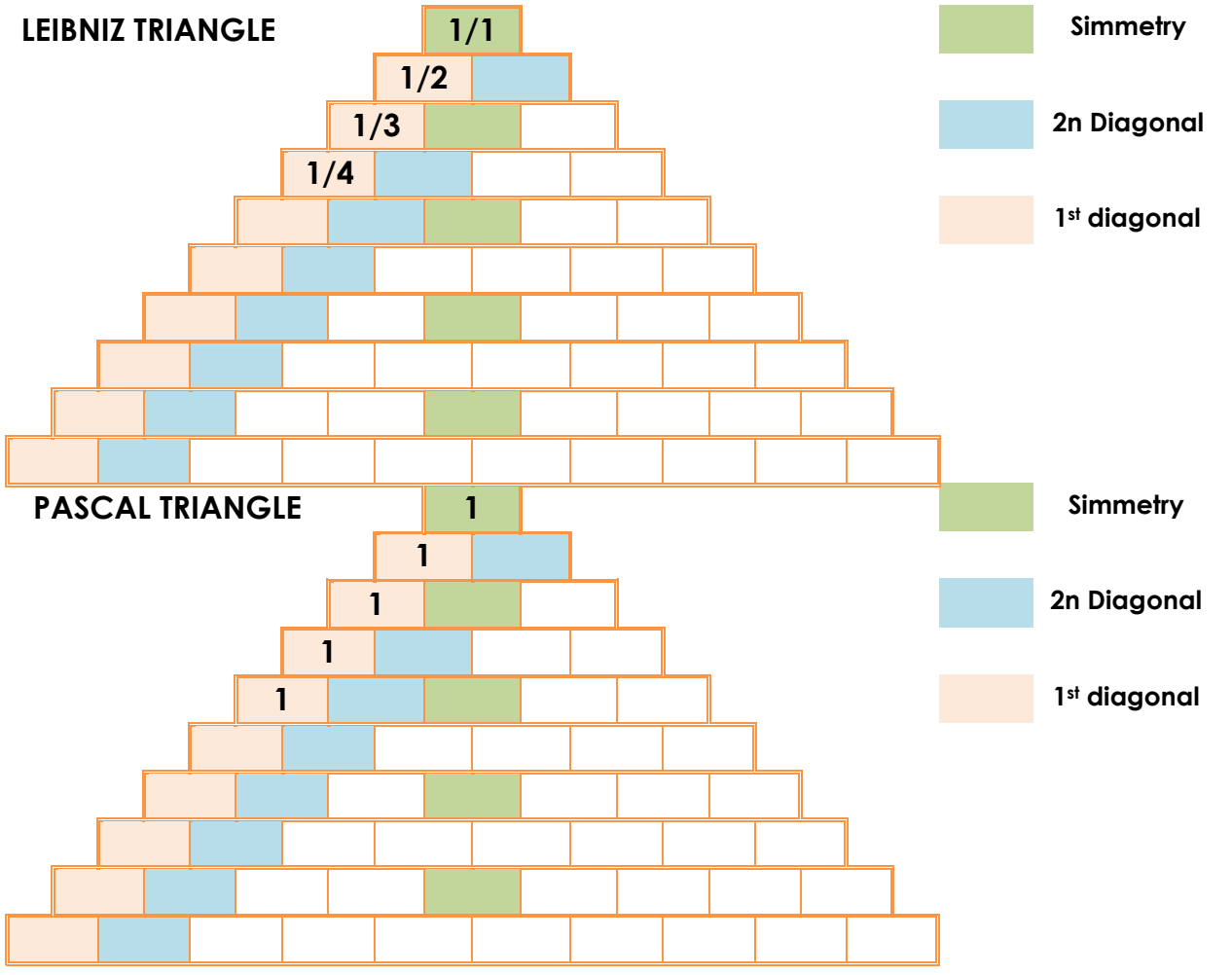
LEIBNIZ TRIANGLE

The Leibniz triangle is a triangular arrangement of fractions in which **the ends of each row consist of a unit fraction**, the reciprocal of the row number while the other numbers are obtained by adding the two numbers immediately **below**. The Leibniz numbers have a **symmetrical** relationship about the vertical line.

PASCAL'S TRIANGLE

Pascal's triangle is a triangular arrangement of numbers. It is generated from the top vertex, the number 1, and to construct further rows we continue to place **1s on the ends** of each row while the other numbers are obtained by adding the two numbers immediately **above**. The Pascal numbers have a **symmetry** relation about the vertical line.

4. Write ten rows of the Pascal and the Leibniz triangle. Check the symmetry relation.



5. Analyse both triangles. Numbers in these triangles have some properties. For example, numbers in the diagonals follow a recurrent rule and can be expressed as a function of the row where they are.
Fill this grid as in the example. The objective is to describe the properties in words and algebraically.

Sequence	Write the sequence in relation to the row	Describe the relationship in words	Algebraically (n=row)
Pascal 2nd diagonal	{1, 2, 3, 4, 5, ...} Row 2 → 1 Row 3 → 2 Row 234 → Row n →	The numbers on the 2nd diagonal are one unit less than the row number. Each number is one unit more than the one on the above row.	n
Leibniz 1st diagonal			
Pascal 3rd diagonal			
Leibniz 2nd diagonal			
Leibniz Sum of a row denominators			
Product of the corresponding terms of both triangles (Pascal and Leibniz)			