A Tutorial on CLIFFORD* with eClifford† and Bigebra‡
A Maple Package for Clifford and Grassmann Algebras

Alterman Conference on Geometric Algebra
and Summer School on Kähler Calculus
Brasov, Romania, August 1–9, 2016
by

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August 1, 2016

Abstract

Various computations in Clifford algebras $C\ell(V,B)$ of an arbitrary bilinear form B in $\dim V \leq 9$ can be performed with a free package CLIFFORD for Maple. Here, the bilinear form B is arbitrary, not necessarily symmetric, or, it could be purely symbolic. Since the package is based on Chevalley's definition of Clifford algebra as a subalgebra of an endomorphism algebra of Grassmann algebra, the underlying basis in $C\ell(B)$ is an undotted Grassmann basis, although a dotted Grassmann basis can be used when the antisymmetric part of B is non-zero. A new experimental package eClifford for $C\ell_{p,q,r}$ uses a different database and extends computations to vector spaces V of arbitrary dimension. Using CLIFFORD, one can solve, for example, algebraic equations when searching for general elements satisfying certain conditions, solve an eigenvalue problem for a Clifford number, and find its minimal polynomial, or compute algebra representations, such as spinor or regular. One can compute with Clifford algebras $C\ell_{p,q}$ viewed as twisted group rings $\mathbb{R}^t[G_{p,q}]$ of Salingaros vee groups $G_{p,q}$. Also, computations with quaternions, split quaternions, octonions, and matrices with entries in a Clifford algebra can easily be completed. Due to the fact that CLIFFORD is a Maple package based on Maple programming language, that is, it runs inside Maple, all Maple packages are available. Thus, CLIFFORD can be easily made to work with new special-purpose packages written by the user. Some examples of algorithms used in the package and computations will be presented.

Keywords: Bigebra, Clifford algebra, CLIFFORD, contraction, dotted wedge product, grade involution, Grassmann algebra, group algebra, twisted group algebra, multivector, octonions, quaternions, reversion, transposition, spinors, vee group, wedge product

Table of Contents (tentative):

*CLIFFORD is a Maple package developed and maintained jointly by R. Abłamowicz and Bertfried Fauser

†eClifford is a Maple package developed and maintained by R. Ablamowicz

‡Bigebra is a Maple package for computing with tensors and Hopf algebras. It has been developed by Bertfried Fauser.

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(a) reversion, grade involution, conjugation, transposition $T_{\varepsilon}\tilde{}$ (in $C\ell_{p,q}$)
(b) spinor representations of $C\ell_{p,q}$
(c) computations with matrices with entries in a Clifford algebra

9. Research with `CLIFFORD` and related packages, such as `SymGroupAlgebra`:
   (a) Deriving and proving properties of the transposition anti-involution $T_{\varepsilon}\tilde{}$ in $C\ell_{p,q}$.
   (b) Computations with Salingaros vee groups $G_{p,q}$
   (c) Relating Clifford algebra $C\ell_{p,q}$ to the group algebra of Salingaros vee group $G_{p,q}$:
      (i) As a homomorphic image of the group algebra $\mathbb{R}[G_{p,q}]$ modulo a two dimensional ideal (Chernov [13])
      (ii) As a twisted group ring (Albuquerque and Majid [14])
10. On parallelizing the Clifford product in `CLIFFORD`
11. Using periodicity theorems in higher dimensional Clifford algebras or using `eCLIFFORD`
12. Appendix:
   (a) Sample help pages
   (b) Sample Maple worksheets created to derive the above papers, go to `http://math.tntech.edu/rafal/publications.html`

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