talk

THE AFFINE AND PROJECTIVE GEOMETRIES FROM GRASSMANN'S POINT OF VIEW

Ramon González [a]

[a] Institut Pere Calders
Campus Universitat Autònoma de Barcelona s/n
08193 Cerdanyola del Vallès, Sapin.
rgonzalezcalvet@gmail.com

Grassmann's powerful but largely undefined approach to affine and projective geometries has roots in Möbius' barycentric calculus [1]. In his appraisal of the former's work, Peano showed explicitly the relation of barycentric coordinates with Cartesian coordinates [2]. In our Treatise ([3] p. 33) we went beyond Peano's work by displaying the advantage of barycentric coordinates in dealing with the main theorems of projective geometry (Desargues, Pappus, etc.). By giving the equations of lines and planes with barycentric coordinates, we explain the geometric duality in a purely algebraic way ([3] p. 43, [4]). The generalization of the barycentric coordinates leads to projective frames and coordinates, which allows us to work with the whole projective geometry of an n-dimensional space without defining the projective space $\mathbb{PR}_n$ as projection of an $n+1$ dimensional space. We will also display the advantages of expressing the equations of quadrics with projective coordinates of the three-dimensional space. According to Grassmann ([5], [6] p. 385), the product of two points is a line, the product of three points is their plane and the product of four points is the whole space. In the same way, the successive products of dual points in the dual space generate geometric elements having decreasing dimensions [7]. Then, Grassmann's products of points and dual points will be identified respectively with the operators *join* and *meet* of the projective geometry. Finally, let us emphasize that all these conclusions can be generalized to n-dimensional spaces.

REFERENCES

[1] A. F. Möbius, *Der Barycentrische Calcul* (Leipzig, 1827). Facsimile edition of Georg Olms Verlag (Hildesheim, 1976).

[2] G. Peano, "Saggio di calcolo geometrico" (1896), *Opere Scelte* III, pp. 166-186, ed. Cremonese, (Roma, 1959). Translated by Hubert C. Kennedy in "Essay on geometrical calculus", *Selected works of Giuseppe Peano*, pp. 169-188, Univ. of Toronto Press (1973).

[3] R. González Calvet, *Treatise of Plane Geometry through Geometric Algebra* (Cerdanyola del Vallès, 2007).

[4] R. González Calvet, *El álgebra geométrica del espacio y tiempo* (2011-) http://www.xtec.cat/ rgonzal1/espacio.htm, p. 64.

[5] H. Grassmann, *Extension Theory* (2000), in the series *History of Mathematics* vol. 19, American Mathematical Society and London Mathematical Society, p. 138. Translation of the 2nd edition of *Die Ausdehnungslehre* (Berlin, 1862) by Lloyd C. Kannenberg.

[6] E. Cartan, "Nombres complexes", *Encyclopédie des Science Mathématiques* (French edition) I, 1, article 1-5, pp. 329-468, Gauthier Villars (Paris, 1908). Reprinted by Jacques Gabay cop. (2005).

[7] R. González Calvet, "How to explain affine point geometry". Talk given at the ICCA 10 (Tartu, August 2014). http://www.xtec.cat/ rgonzal1/affine_point_geometry.pdf .