talk

INVERSE KINEMATICS BASED ON BINOCULAR VISION

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Our goal is to grasp an object lying in 3D space using inverse kinematics. Using image analysis with two cameras located in independent axes we calculate the position of the reference line and grasp the object lying in this line. We use some advantages of CGA in our particular setting.

Classically, for modeling a 3D robot, the whole CGA (i.e. $\mathscr{C}l(4,1)$) is used, where the embedding $\mathbb{R}^3 \to \mathbb{K}^4 \to \mathbb{R}^{4,1}$ is considered, where $\mathbb{K}^4$ is a null cone and $\mathbb{R}^{4,1}$ is a Minkowski space. Consequently, the embedding is of the form $c(x) = x + \frac{1}{2}x^2 e_\infty + e_0$. Note that e_0 and e_∞ play the role of the origin and the infinity, respectively.

CGA provides a set of basic geometric entities to compute with, namely points, spheres, planes, circles, lines, and point pairs. These entities have two algebraic representations, IPNS and OPNS which are duals of each other by convolution $(\cdot)^*$. In OPNS representation we can handled with spheres $c(x_1) \wedge c(x_2) \wedge c(x_3) \wedge c(x_4)$, planes $c(x_1) \wedge c(x_2) \wedge c(x_3) \wedge e_\infty$, circles $c(x_1) \wedge c(x_2) \wedge c(x_3)$ and lines $c(x_1) \wedge c(x_2) \wedge e_\infty$. In OPNS, the sphere is represented as $S = c(x) - \frac{1}{2}r^2 e_\infty$, where $c(x)$ is a center point and r is a radius. Note that the properties and definitions of conformal geometric algebras can be found in e.g. [2].

The problem, we split into two parts:

1. Binocular vision. Using the data obtained from both cameras, the line symmetry is transformed from the image coordinates to camera coordinates, i.e. from $\mathbb{R}^2 \to \mathbb{R}^3$. With the lines L_1 and L_2 in IPNS representation and the camera focuses c_1 and c_2 we create two planes $L_1 \wedge c_1$ and $L_2 \wedge c_2$ in IPNS, such that, the reference line is an intersection of these planes.

1. Inverse kinematics. Our robot has five degrees of freedom obtained by means of the five joint angles $\theta_1, \dots, \theta_5$. Our goal is to find the joint angles in terms of the target position. In CGA, this inverse kinematics problem can be solved in a intuitive way by the handling of intersections of spheres. For example, let us have two links $P_{i-1}P_i$ and P_iP_{i+1}, i.e. the joint P_i has to lie on the sphere with center point P_{i-1} and on the sphere with center point P_{i+1} with the length of the vectors $\overrightarrow{P_{i-1}P_i}$ and $\overrightarrow{P_iP_{i+1}}$ as the radius respectively. The first sphere is represented by OPNS element $S_{i-1} = P_{i-1} - \frac{1}{2}|\overrightarrow{P_{i-1}P_i}|^2 e_\infty$ and the second by OPNS element $S_i = P_{i+1} - \frac{1}{2}|\overrightarrow{P_iP_{i+1}}|^2 e_\infty$. From the theory it is easy to see that $(S_{i-1} \wedge S_i)^*$ is an OPNS representation of circle, such that P_i belongs to this circle.

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